Transformation Optics
& the Control of Electromagnetic Radiation

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Abstract: Ray optics gives us some control over propagation of light, but fails to account for the wave nature of light. Even more spectacularly it has nothing at all to say about controlling the so called 'near field'. In contrast the new technique of transformation optics offers the possibility of complete control of radiation, correct to the level of Maxwell's equations. Using this technology we can specify the material parameters needed to arrange the electric fields, magnetic fields and the Poynting vector almost as we choose.

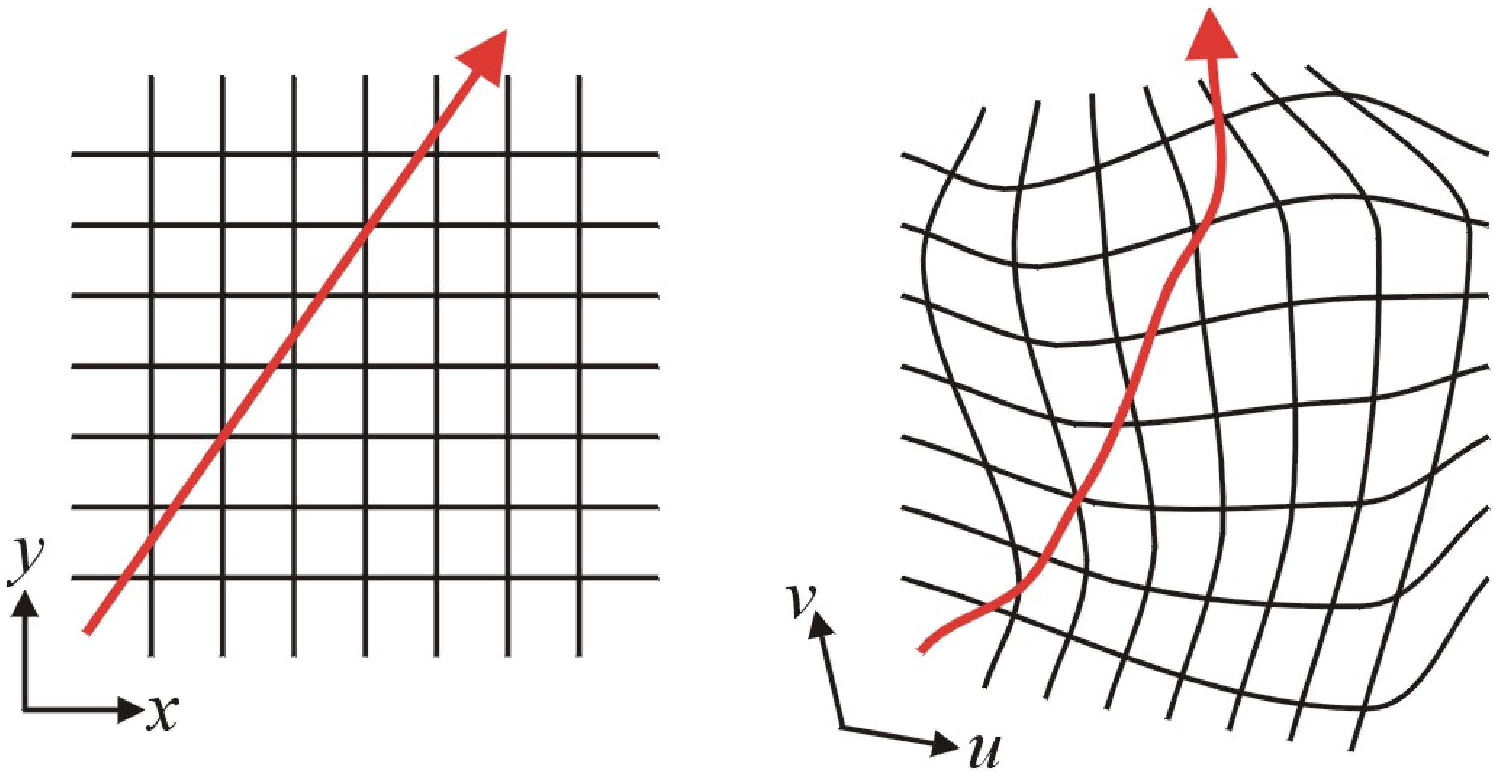
D. Schurig, J.B. Pendry, D.R. Smith, *Optics Express* **14**, 9794 (2006).

J.B. Pendry, *Contemporary Physics* **45,** 191 (2004).

J.B. Pendry in *"Coherence and Quantum Optics IX"*, ed. P. Bigelow, J.H. Eberly and C.R. Stroud, Jr. (OSA Publications), pp. 42-52. (2009).

Controlling Electromagnetic Fields

Exploiting the freedom of design which metamaterials provide, we show how electromagnetic fields can be redirected at will and propose a design strategy. The conserved fields: electric displacement field, **D**, magnetic induction field, **B**, and Poynting vector, **S**, are all displaced in a consistent manner *and can be arranged at will by a suitable choice of metamaterials*.In general we require materials that are anisotropic and spatially dispersive.

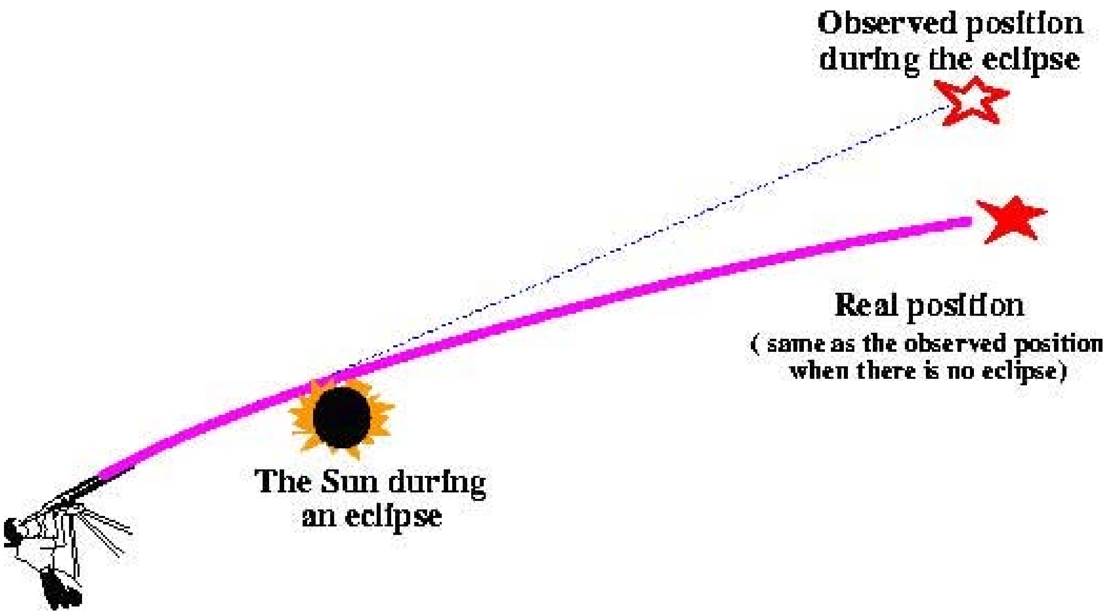


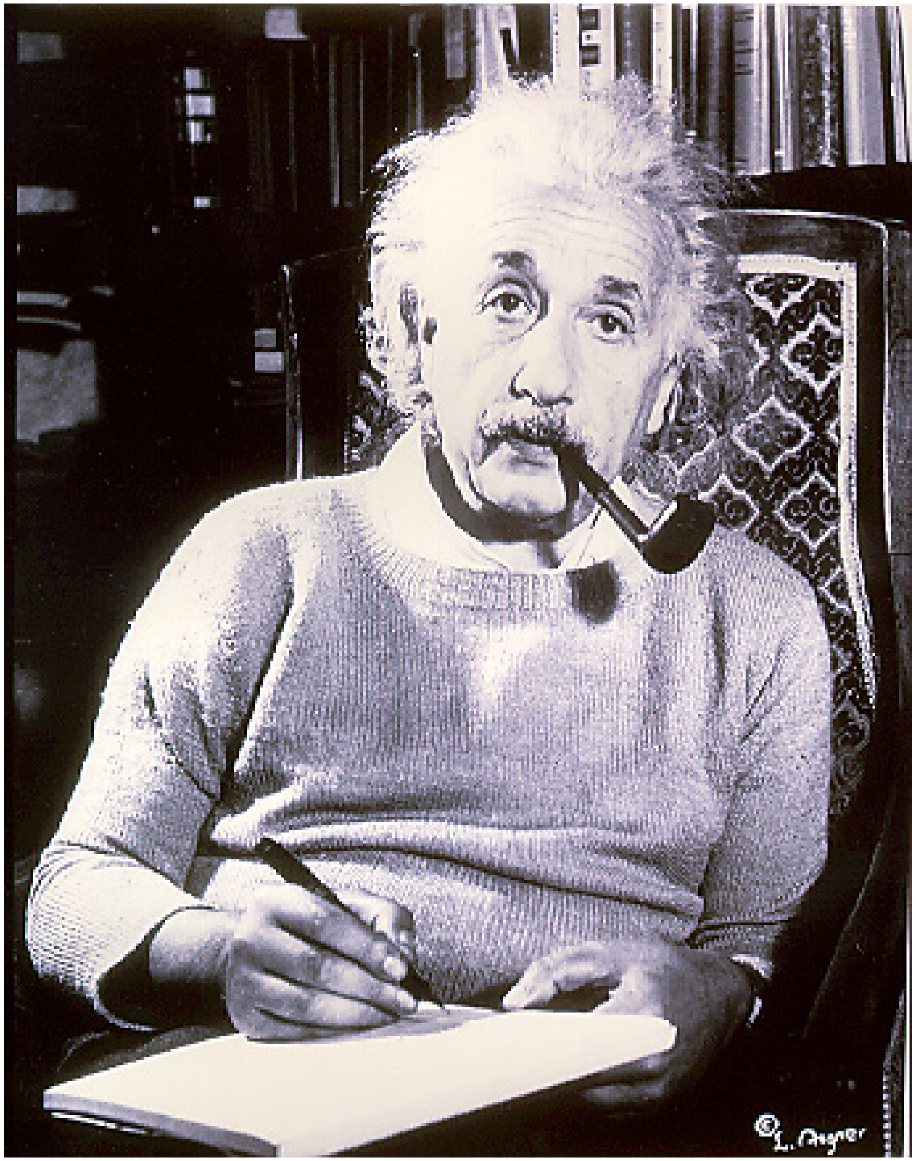
Left: a field line in free space with the background Cartesian coordinate grid shown. Right: the distorted field line with the background coordinates distorted in the same fashion.

Einstein, Light, and Geometry – the theory

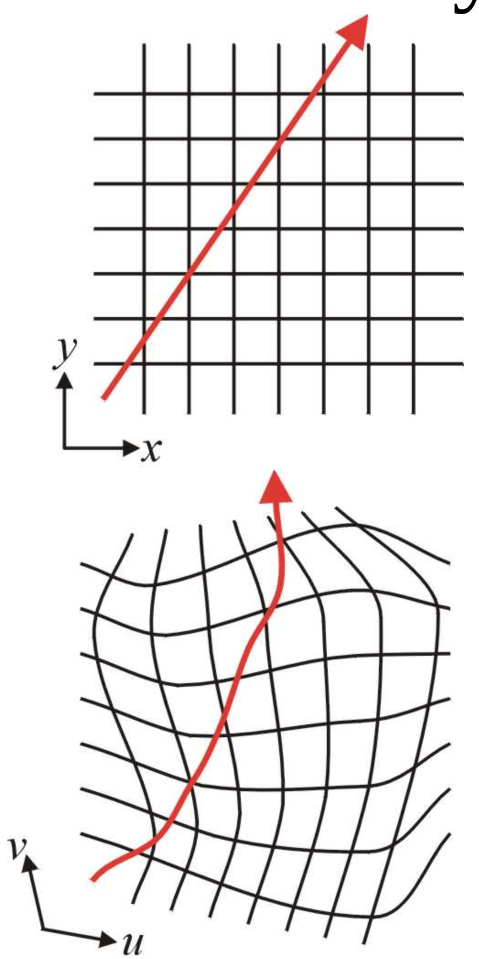
The general theory of relativity: gravity changes geometry.

Therefore gravity should bend light





Formal theory: A. J. Ward and J. B. Pendry, *J Mod Op*, **43** 773 (1996)



Top: a ray in free space with the background Cartesian coordinate grid shown. **Bottom**: the distorted ray trajectory with distorted coordinates.

New coordinates in terms of the old:

$$u(x,y,z),\ v(x,y,z),\ w(x,y,z)$$

In the new coordinate system we must use renormalized values of the permittivity and permeability ()_assuming orthogonal:

$$\tilde{\varepsilon}_u = \varepsilon_u \frac{Q_u Q_v Q_w}{Q_u^2}, \qquad \tilde{\mu}_u = \mu_u \frac{Q_u Q_v Q_w}{Q_u^2}, \qquad \text{etcetera}$$

where,

$$Q_u^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2$$

$$Q_v^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2$$

$$Q_w^2 = \left(\frac{\partial x}{\partial w}\right)^2 + \left(\frac{\partial y}{\partial w}\right)^2 + \left(\frac{\partial z}{\partial w}\right)^2$$

The Transformations

If the distorted system is described by a coordinate transform $x'^{j'}\left(x^{j}\right)$ we define,

$$\Lambda_{j}^{j'} = \frac{\partial x^{j'}}{\partial x^{j}}$$

Then in the new coordinate system we must use modified values of the permittivity and permeability to ensure that Maxwell's equations are satisfied,

$$\varepsilon'^{i'j'} = \left[\det\left(\Lambda\right)\right]^{-1} \Lambda_{i}^{i'} \Lambda_{j}^{j'} \varepsilon^{ij}$$

$$\mu'^{i'j'} = \left[\det\left(\Lambda\right)\right]^{-1} \Lambda_{i}^{i'} \Lambda_{j}^{j'} \mu^{ij}$$

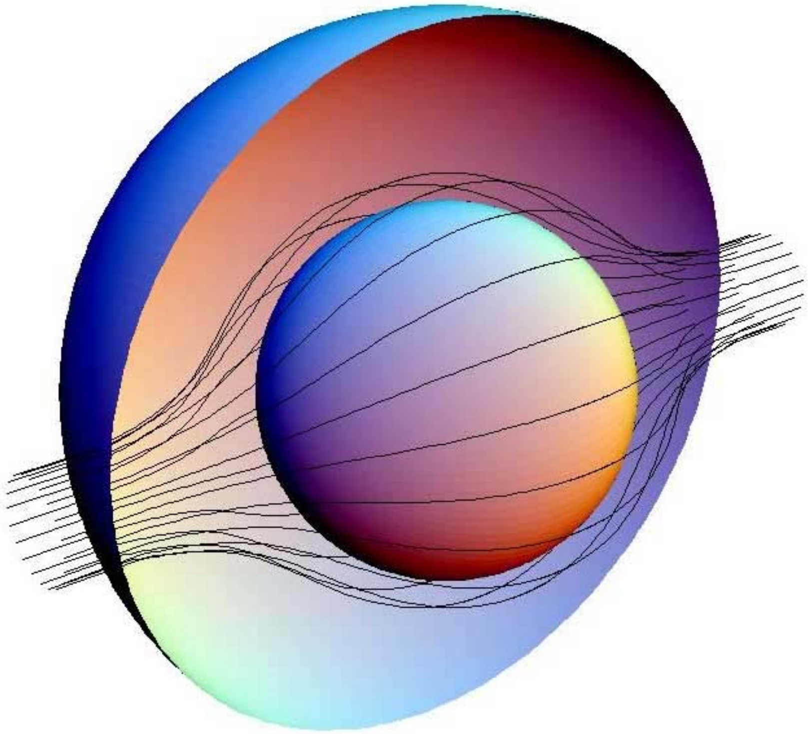
see: D.M. Shyroki http://arxiv.org/abs/physics/0307029v1 (2003)

Cloaking Static Magnetic Fields

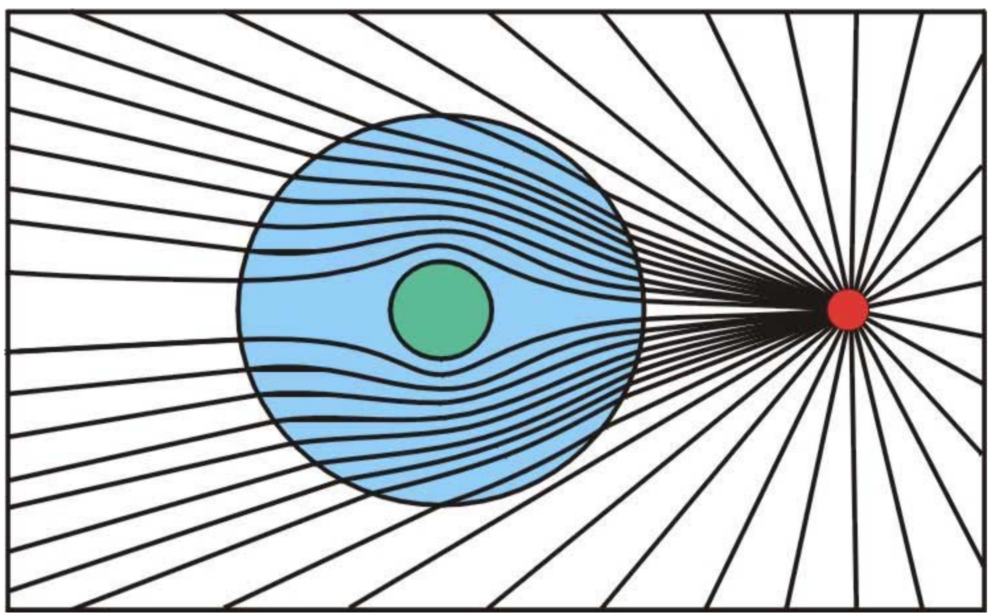
see: J. Phys.: Condens. Matter **19** (2007) 076208 (*B. Wood and JB Pendry*)

Cloaking works for fields as well as waves!

deflection of rays
(far field)

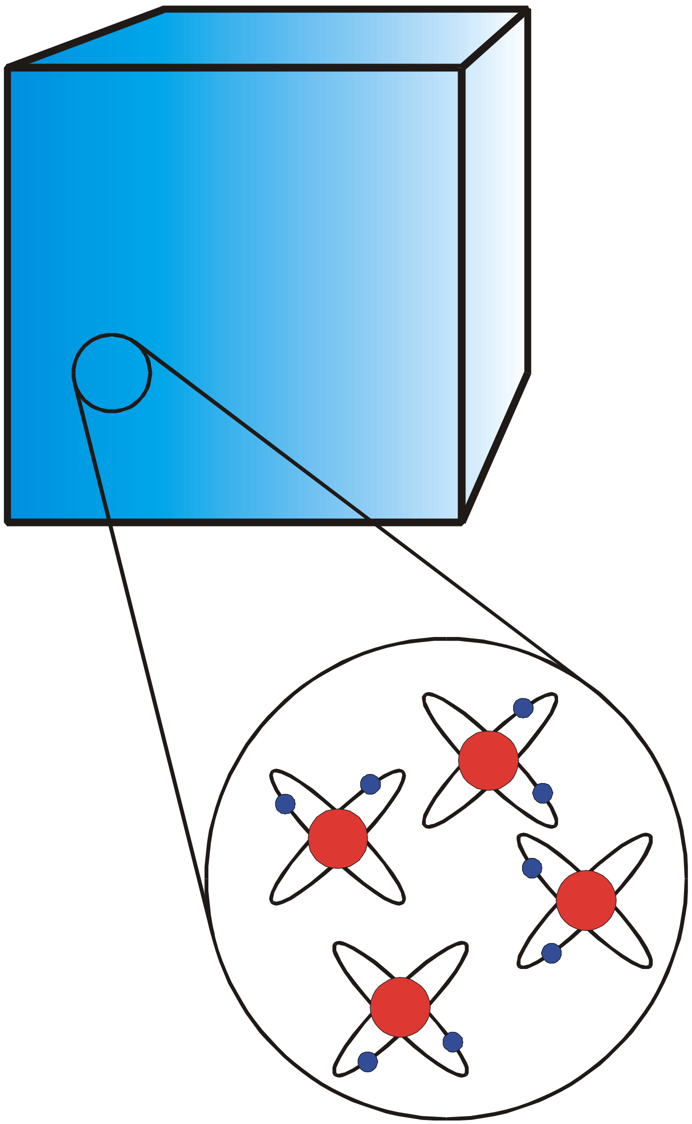


deflection of field lines
(near field)

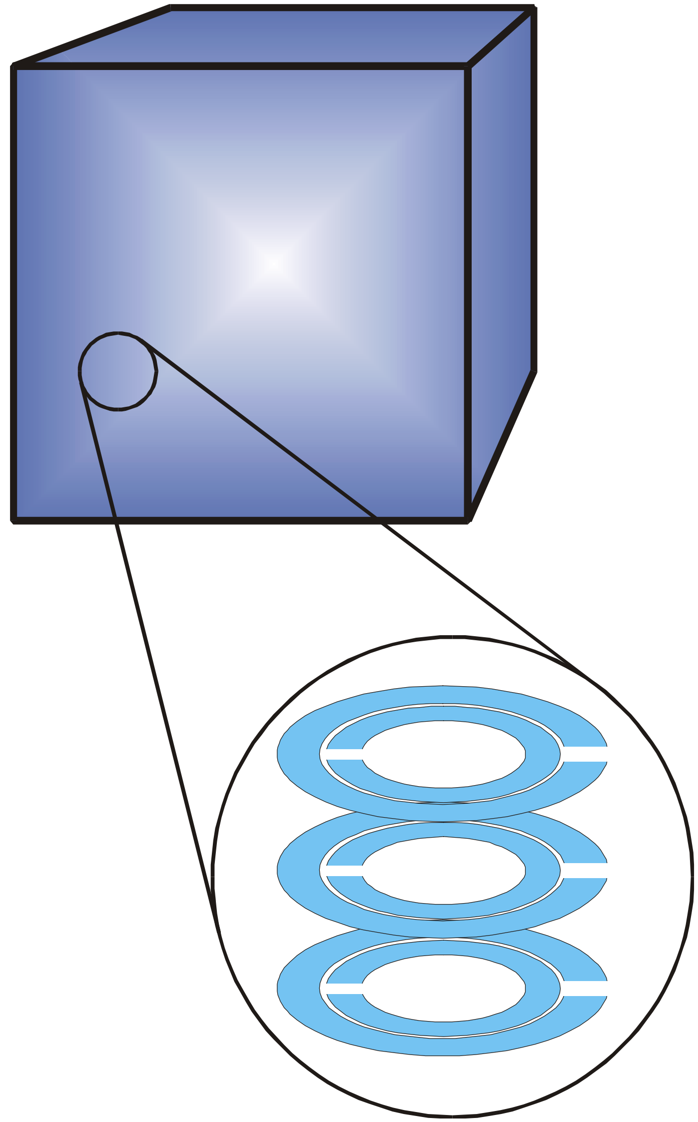


What is a ‘metamaterial’

Conventional materials: properties derive from their constituent *atoms*.



Metamaterials: properties derive from their constituent *units*. These units can be engineered as we please.



What is special about $\omega = 0$?

Electric and magnetic fields decouple – we can assume a pure magnetic field and concentrate on engineering μ.

However, from above, $0 < \mu_\perp < 1$ when we squash the coordinates and since,

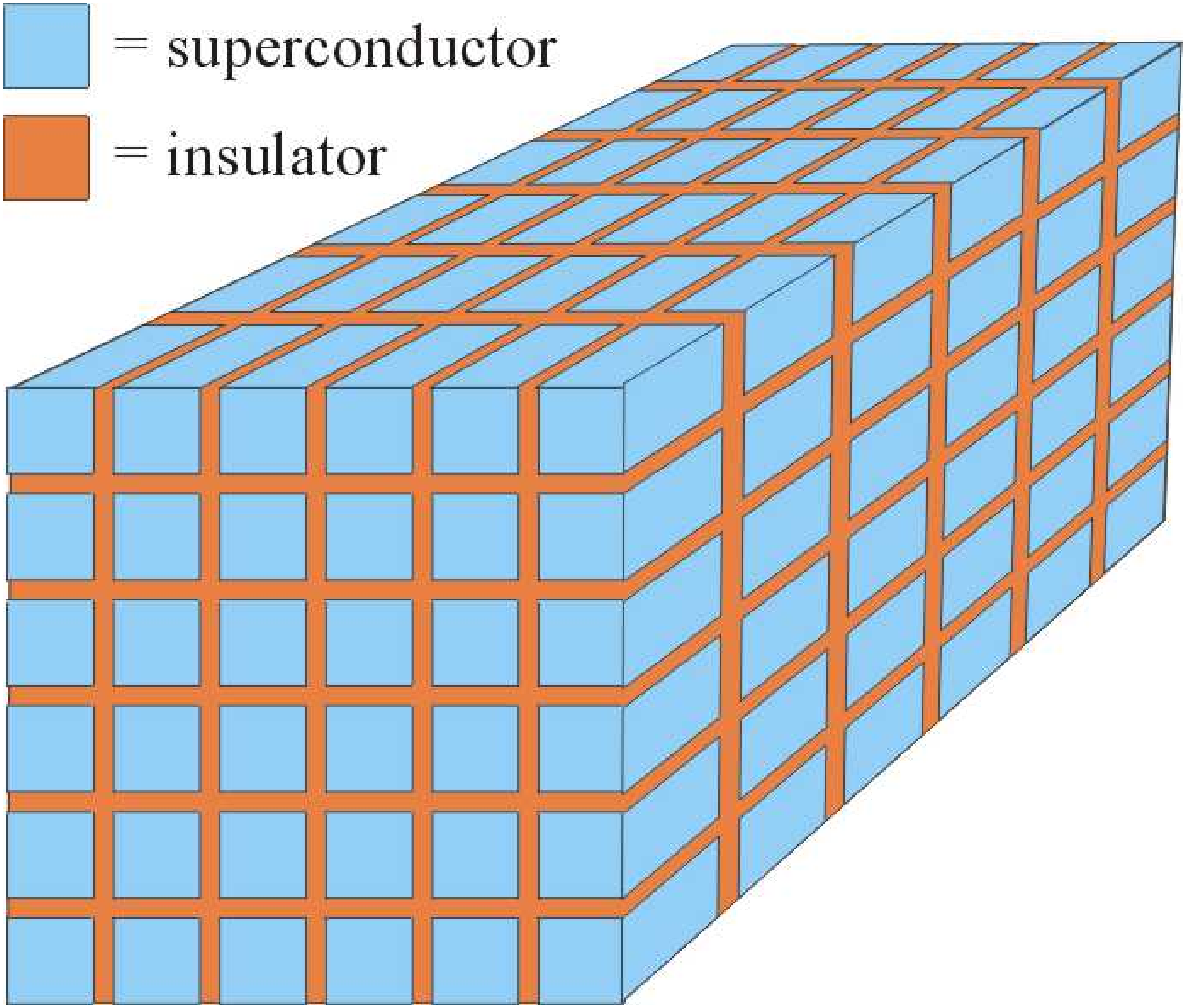
$$\lim_{\omega \to 0} v_{group} = v_{phase} = c_0 \Big/ \sqrt{\varepsilon_{||} \mu_\perp}$$

we can expect problems unless we do something about $\varepsilon_{||}$. *We must engineer both* $\varepsilon_{||}$ *and* $\mu_\perp$.

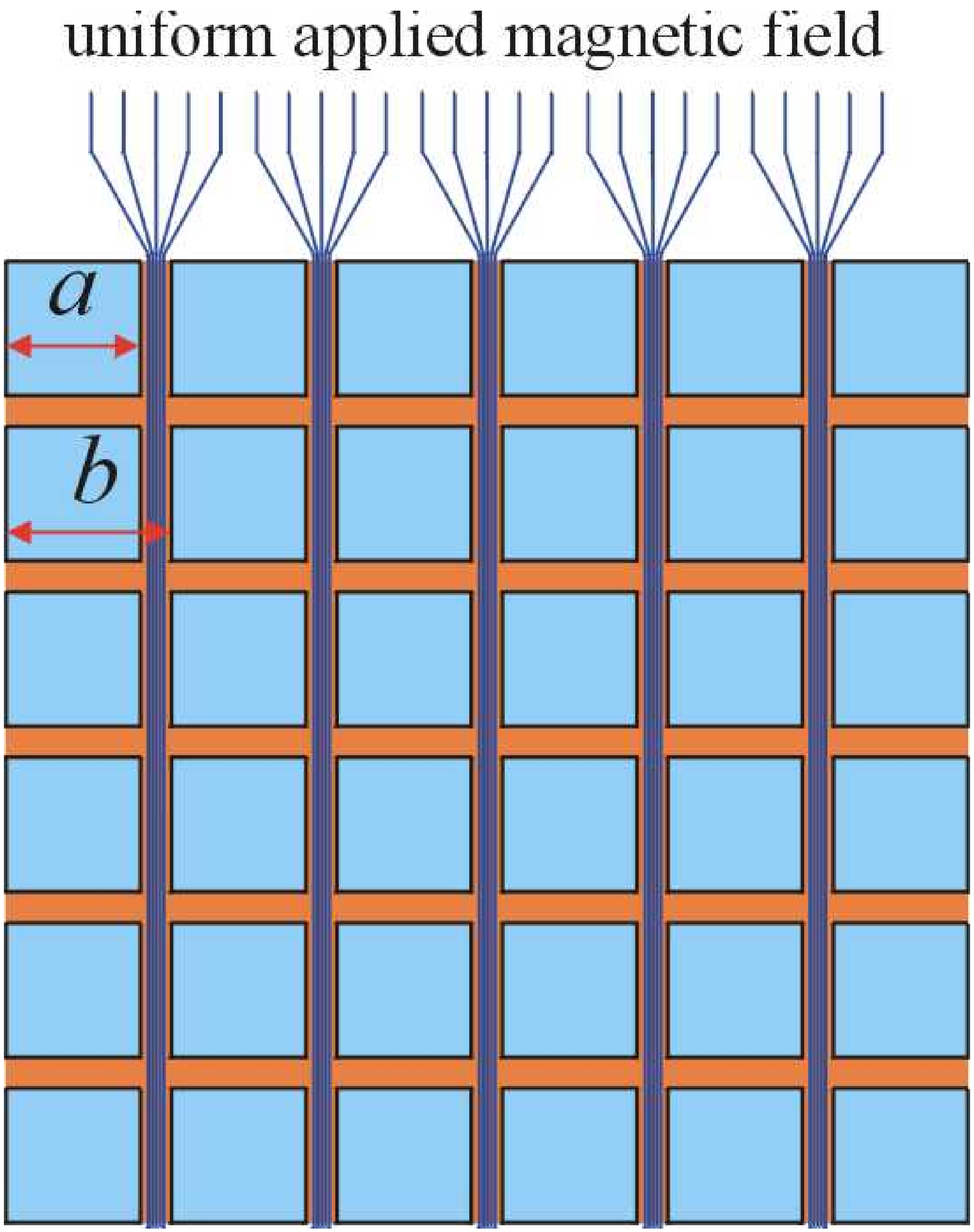
In this way we can achieve $0 < \mu_\perp < 1$ using superconductors.

The electric equivalent of $0 < \varepsilon_\perp < 1$ is not possible. Hence DC magnetic fields can be cloaked but DC electrical fields cannot.

A metamaterial with $0 < \mu < 1$ *and* no dispersion



Calculating μ_{eff}



The effective permeability is given by:

$$\mu_{eff} = \frac{B_{ave}}{\mu_0 H_{ave}}$$

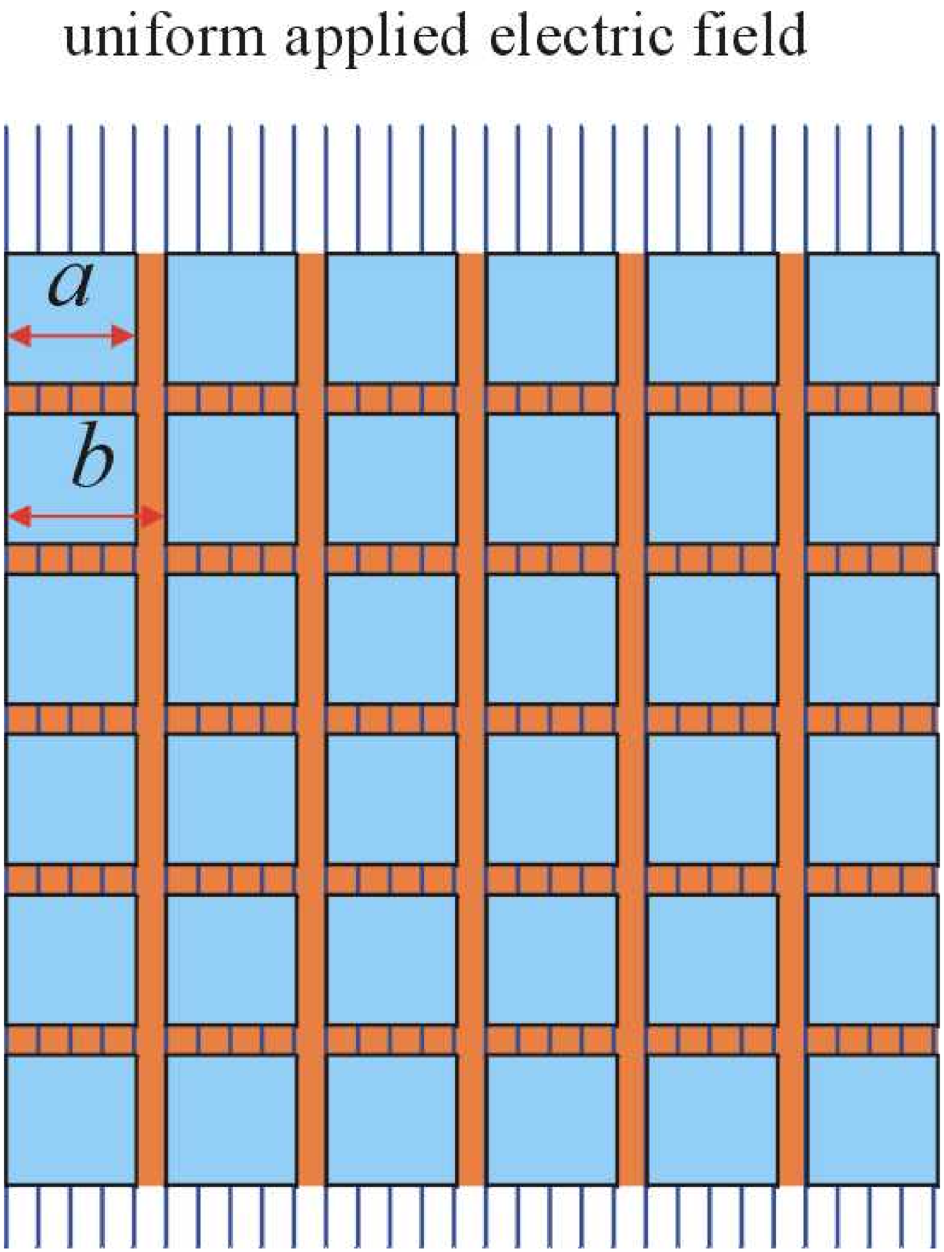
where we define B_{ave} as an average over the cross sectional area of the sample, and H_{ave} as an average along a line through the interstices of the sample. Since all flux is excluded from the superconductor the flux lines are squeezed into a smaller area, hence,

$$\mu_{eff} = \frac{b^2 - a^2}{b^2}$$

Note that this result is independent of the frequency. Does this imply

$$c = c_0 \big/ \sqrt{\mu_{eff}} > c_0?$$

Calculating ε_{eff}



The effective permittivity is given by:

$$\varepsilon_{eff} = \frac{D_{ave}}{\varepsilon_0 E_{ave}}$$

where we define D_{ave} as an average over the cross sectional area of the sample, and E_{ave} as an average along a line through the sample. Since all the E field is concentrated in the gap between superconductors,

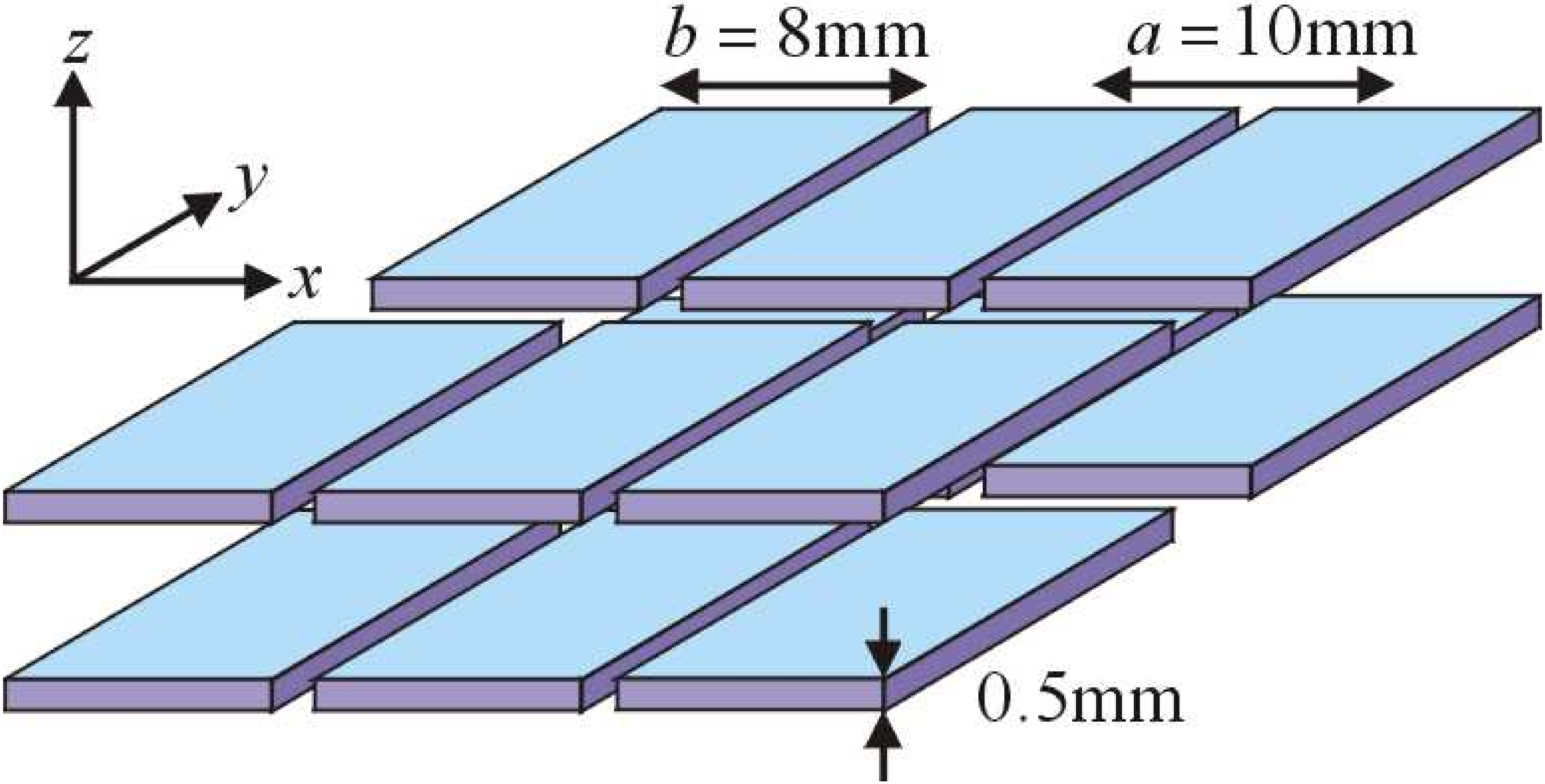
$$\varepsilon_{eff} = b/b - a$$

Hence,

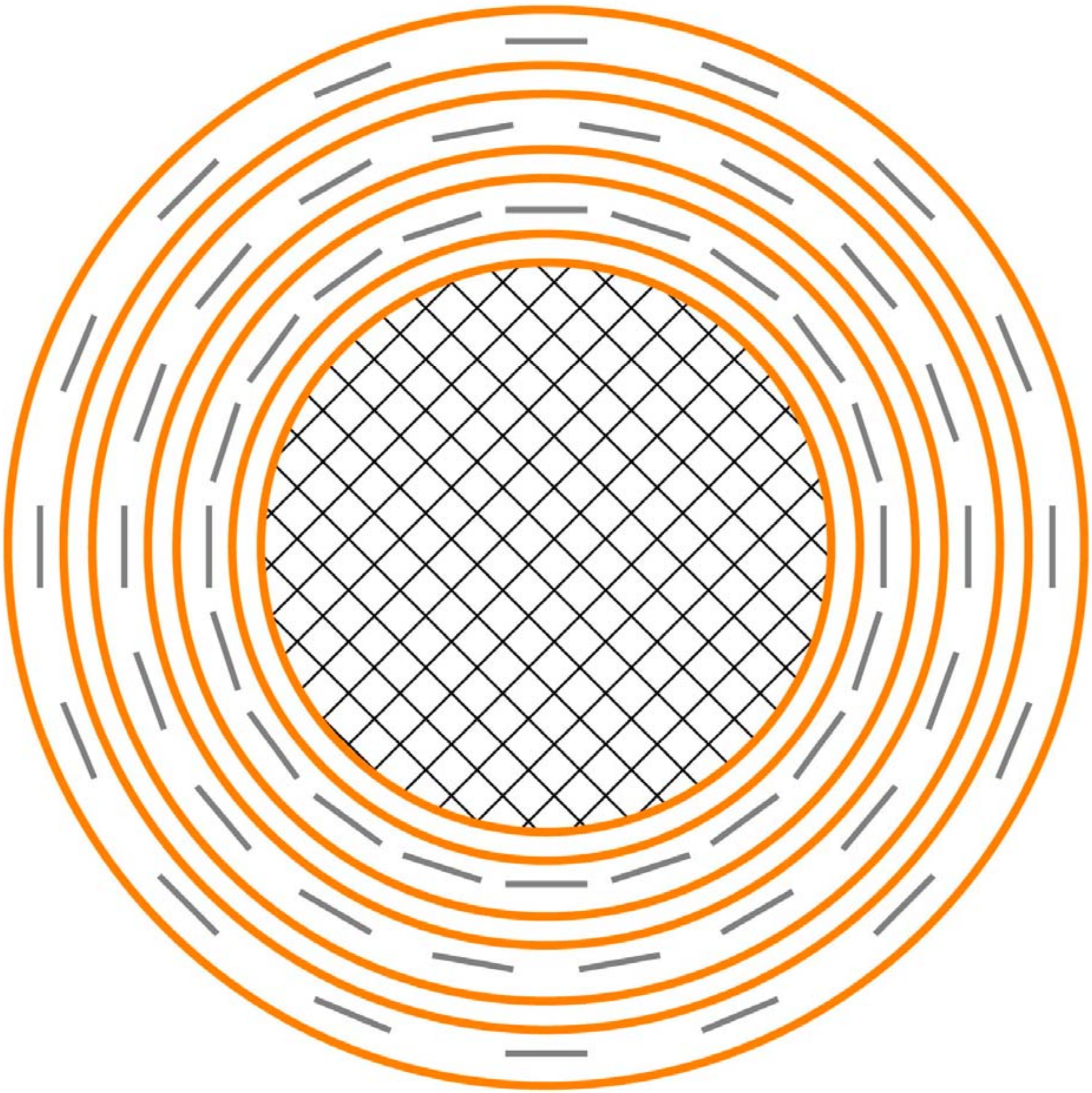
$$\varepsilon_{eff}\mu_{eff} = \frac{b}{b-a}\frac{b^2 - a^2}{b^2} = \frac{b+a}{b} > 1$$

$$\lim_{a \to b} c = \lim_{a \to b} c_0 \Big/ \sqrt{\varepsilon_{eff}\mu_{eff}} = c_0 \Big/ \sqrt{2}$$

Lattice of superconducting plates



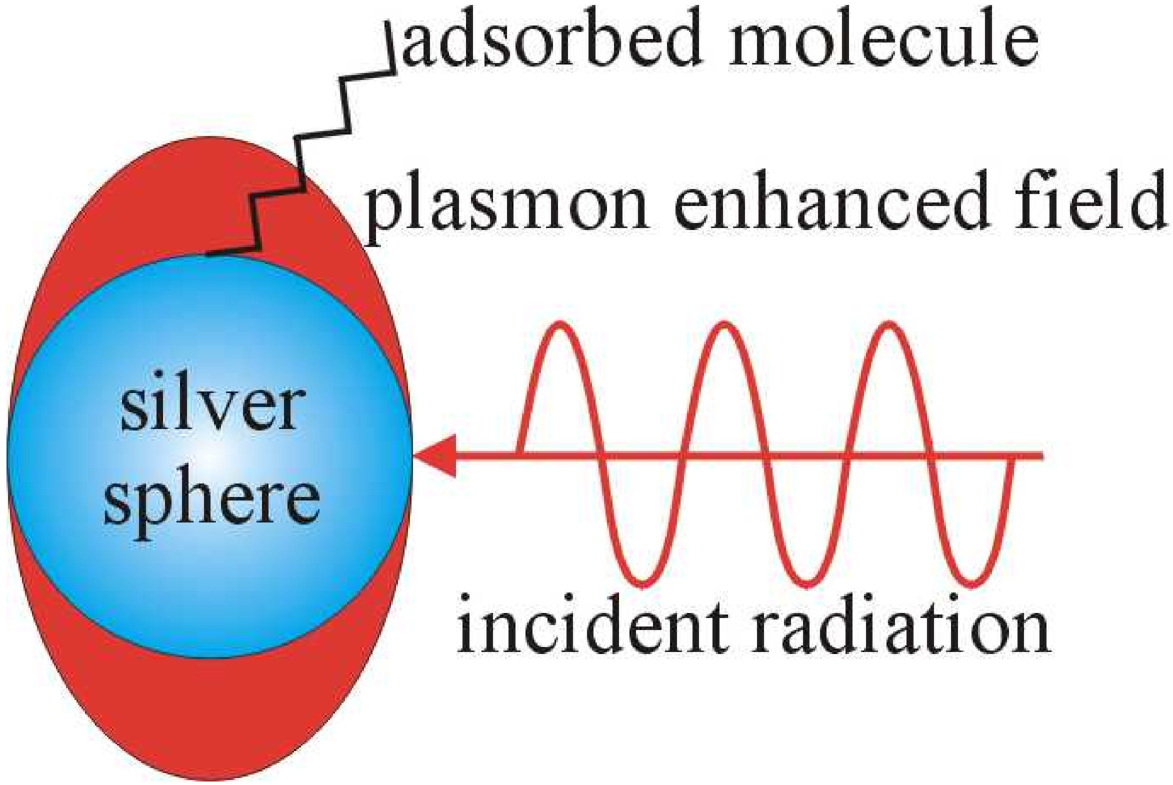
The proposed magnetic cloak



The shaded region in the centre is hidden from external magnetic fields. The plates form broken circles (in cross section); the full circles show the ferrite or amorphous metal.

Detection of Single Molecules

A plasmon resonance in a silver sphere greatly enhances the spectroscopic signature of adsorbed molecules



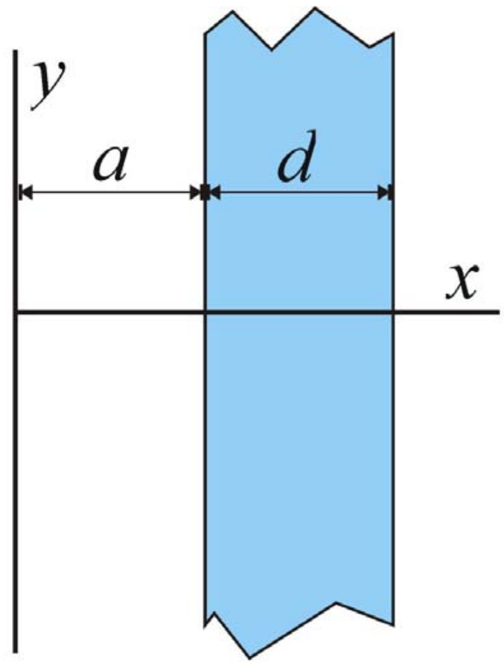
problem: the narrow plasmon resonance restricts the sensitivity to a narrow range of frequencies.

Constructing a broadband absorber

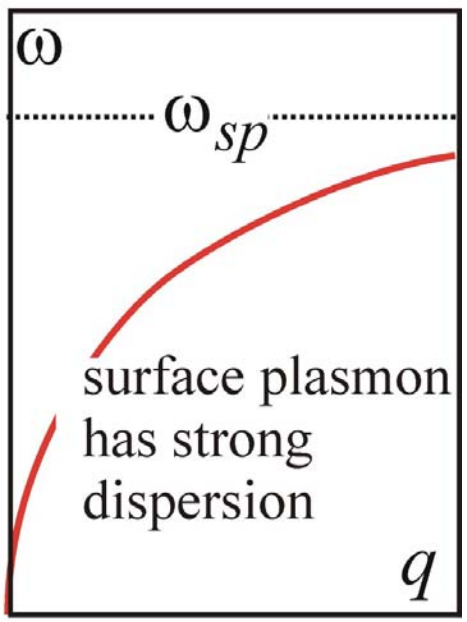
Resonant systems, such as silver spheres, enhance the absorption of radiation hence greatly improving the sensitivity to adsorbed molecules; but absorption by a single resonance is narrow band and therefore of limited use.

- start with a dipole exciting an infinite system – most infinite systems have a *broadband* continuum
- invert about the origin to convert to a finite system excited by a plane wave. The *spectrum is unchanged and remains broadband.*

a metallic slab of finite thickness has a broadband spectrum



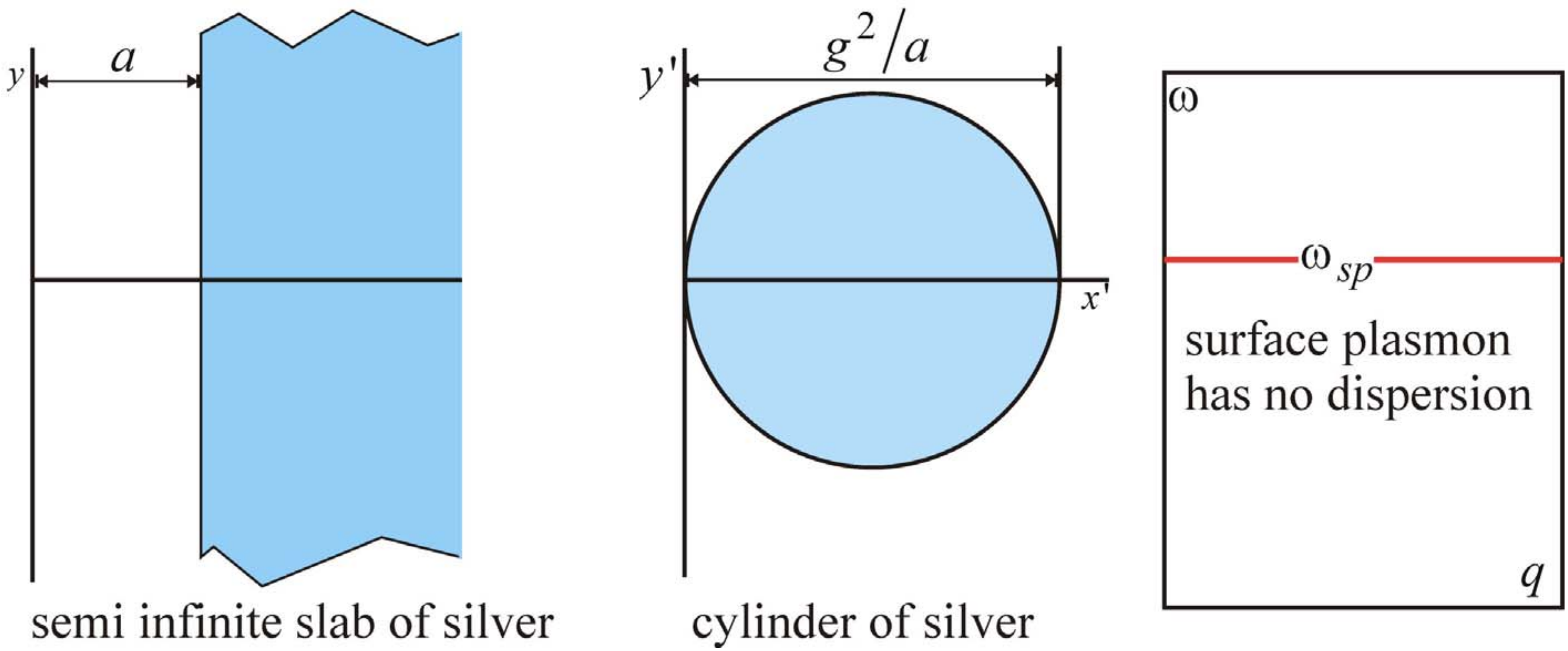
finite slab of silver



Conformal transformations of 2D sub-wavelength objects – electrostatic limit

e.g. inversion about the origin: $z' = 1/z, \quad z = x + iy, \quad \phi(z) \to \phi(z')$

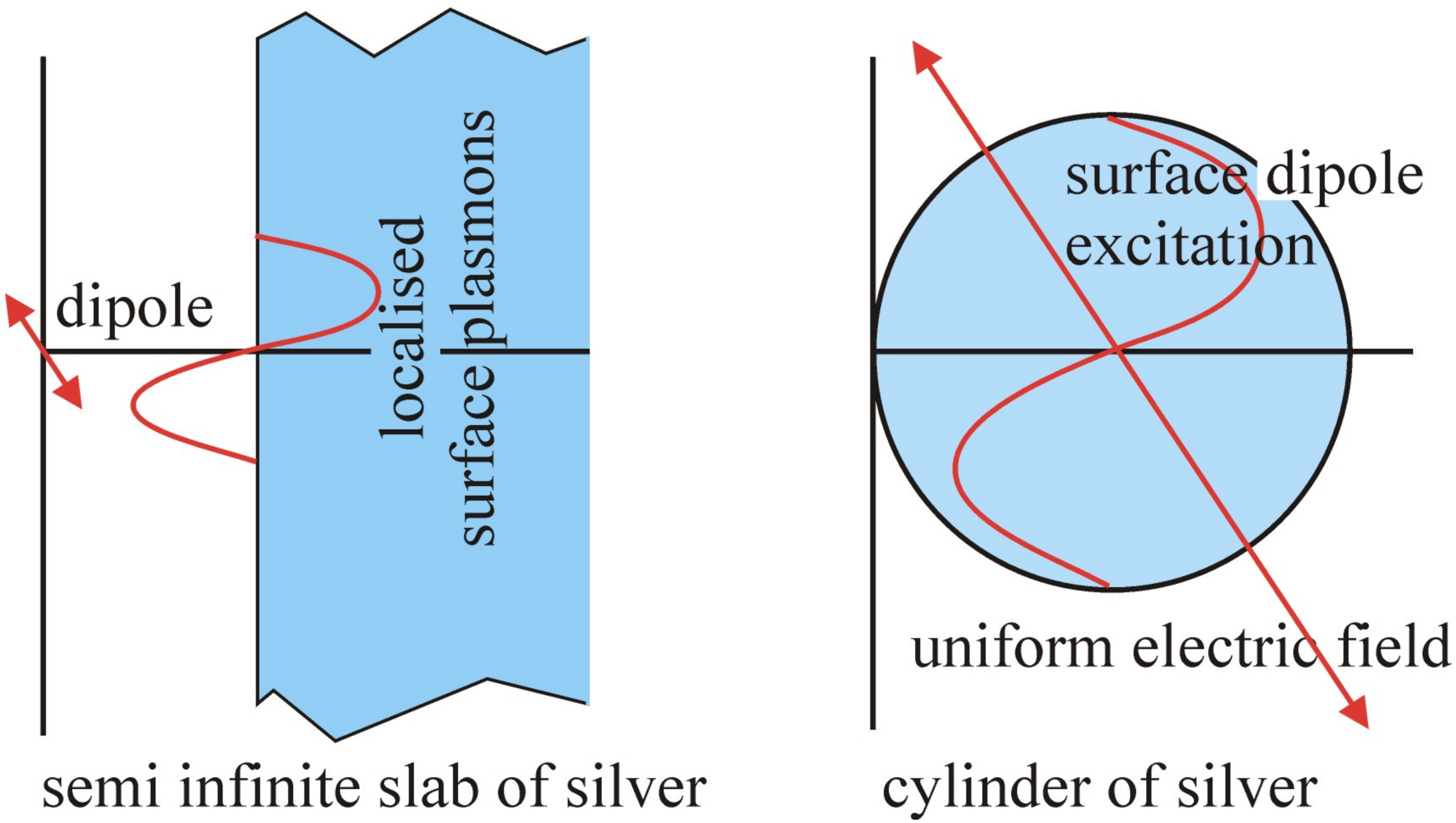
this conformal transformation maps a semi ∞ slab into a cylinder



spectra of the slab and the cylinder are identical in the electrostatic limit

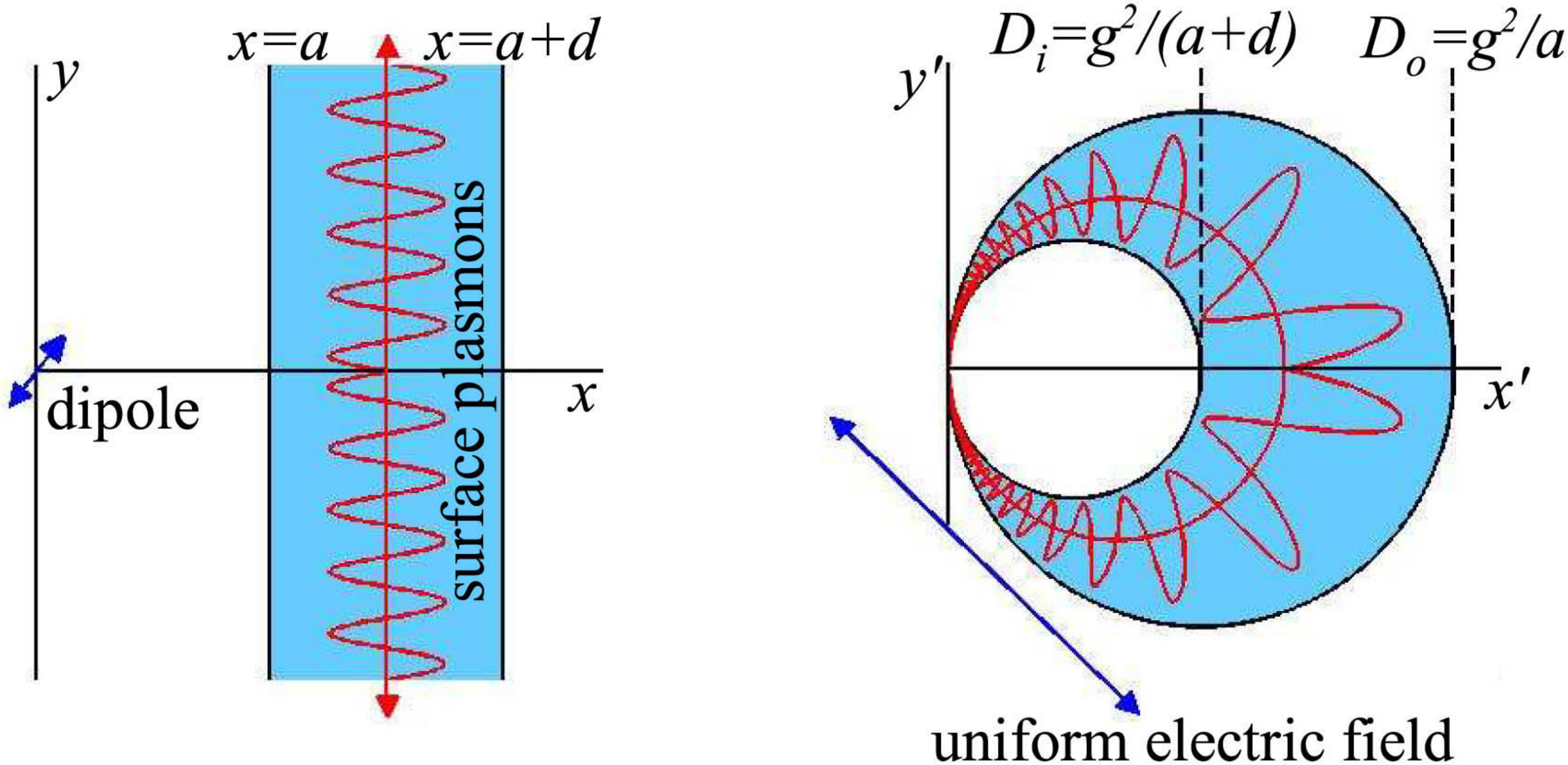
linear momentum, *q*, maps into angular momentum, *m*.

An inversion maps a dipole into a uniform electric field



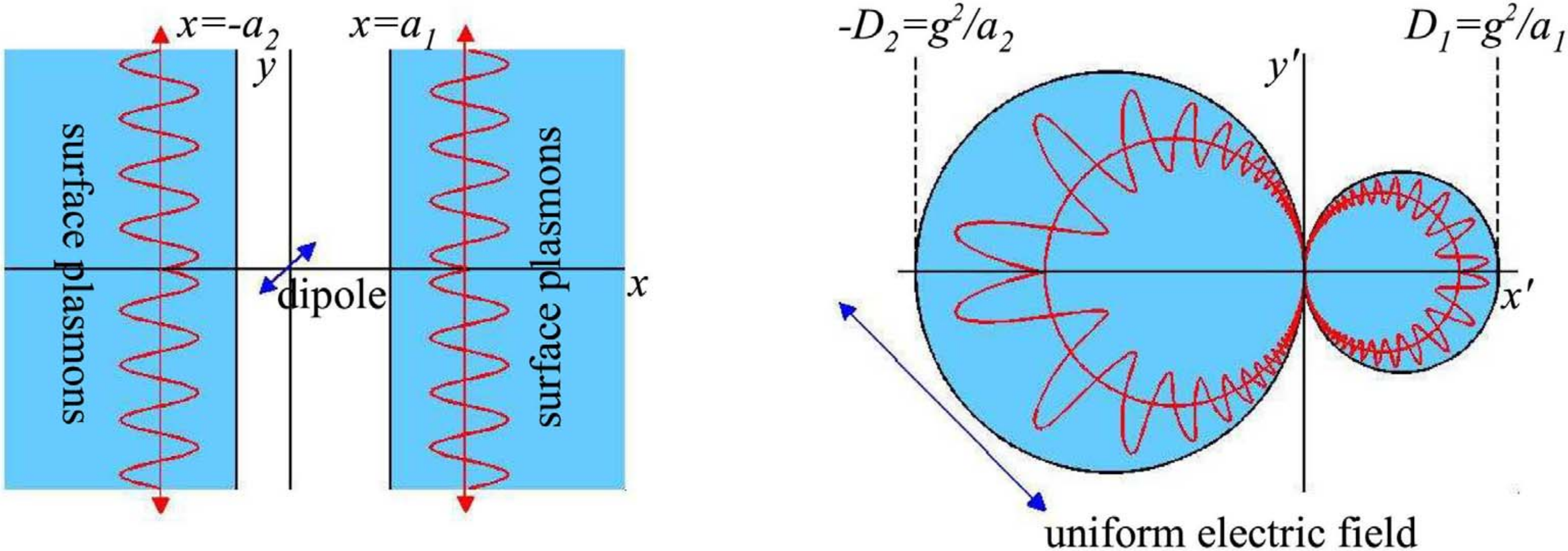
Studying a dipole interacting with a flat surface tells us how a cylinder responds to a plane wave. (We assume the diameter to be sub-wavelength). The absorption cross section comprises a sharp resonance at ω_{sp} in both cases.

Inversion about the origin, $z' = 1/z$, converts a slab to a cylindrical crescent *(Alexandre Aubry, Dang Yuan Lei, Antonio I. Fernandez-Dominguez, et al.)*



Left: a thin slab of metal supports surface plasmons that couple to a dipole source, transporting its energy to infinity. The spectrum is continuous and broadband therefore the process is effective over a wide range of frequencies.
Right: the transformed material now comprises a cylinder with cross section in the form of a crescent. The dipole source is transformed into a uniform electric field.

Inversion about the origin, $z' = 1/z$,
converts two slabs into two kissing cylinders
(Alexandre Aubry, Dang Yuan Lei, Antonio I. Fernandez-Dominguez, et al.)

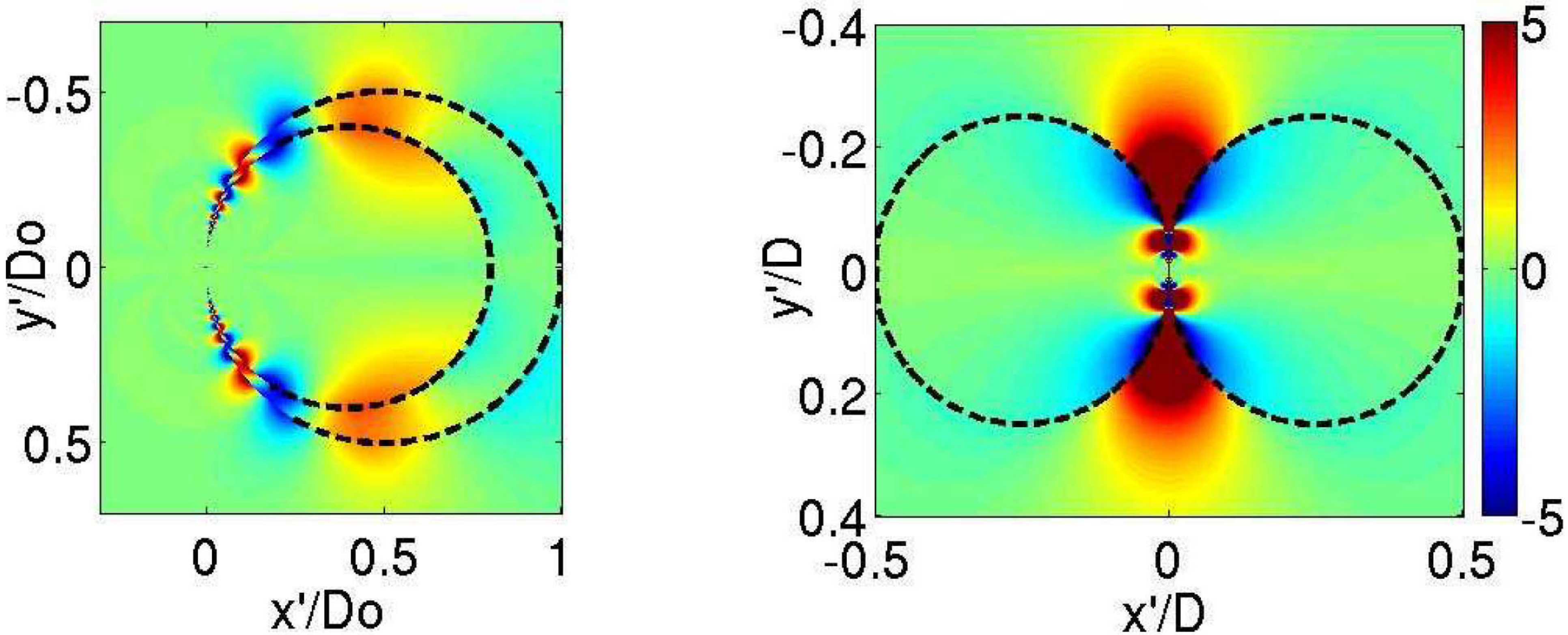


Left: two semi-infinite metallic slabs separated by a thin dielectric film support surface plasmons that couple to a dipole source, transporting its energy to infinity. The spectrum is continuous and broadband therefore the process is effective over a wide range of frequencies.

Right: the transformed material now comprises two kissing cylinders. The dipole source is transformed into a uniform electric field.

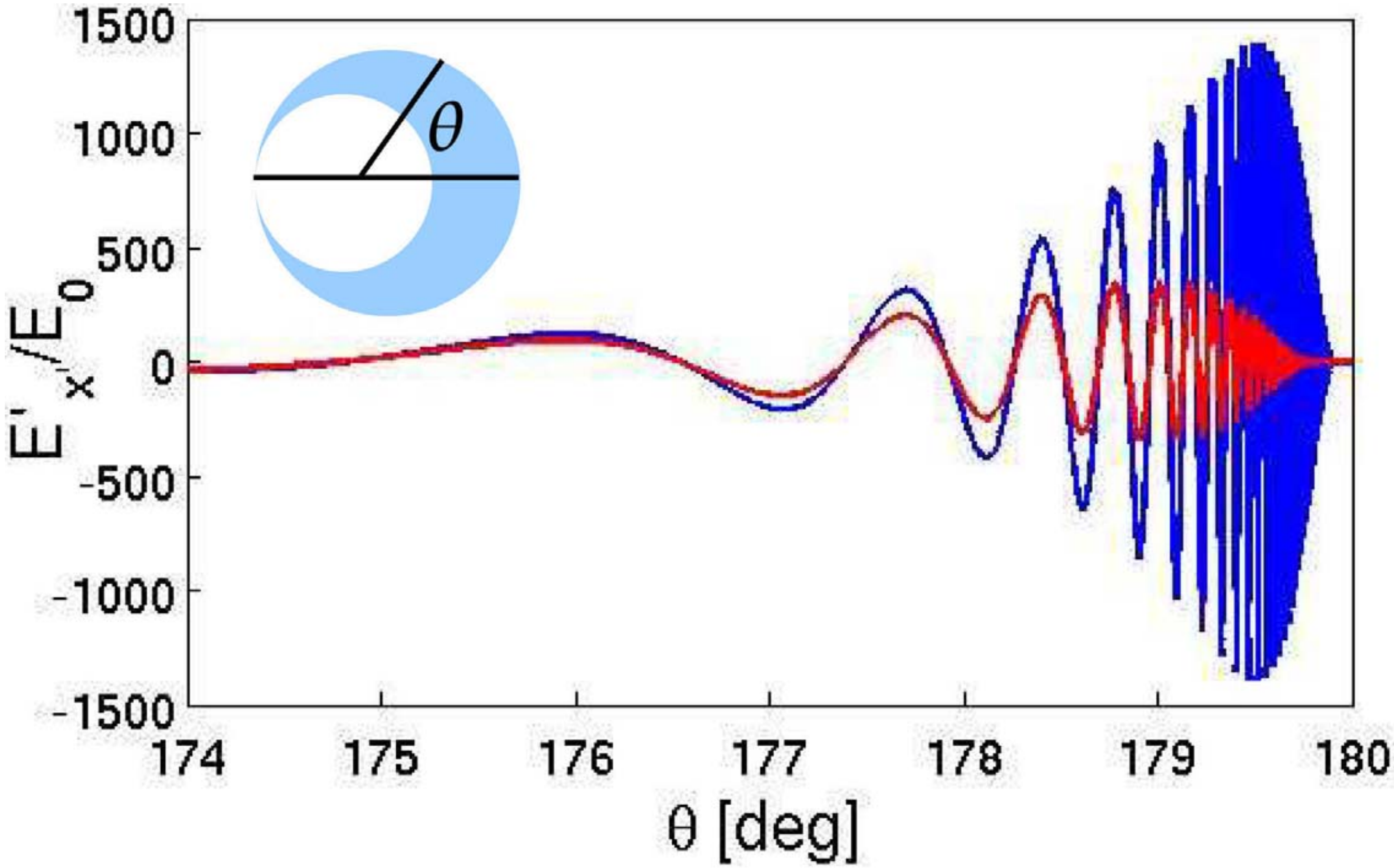
Broadband field enhancement in singular structures

(Alexandre Aubry, Dang Yuan Lei, Antonio I. Fernandez-Dominguez, et al.)



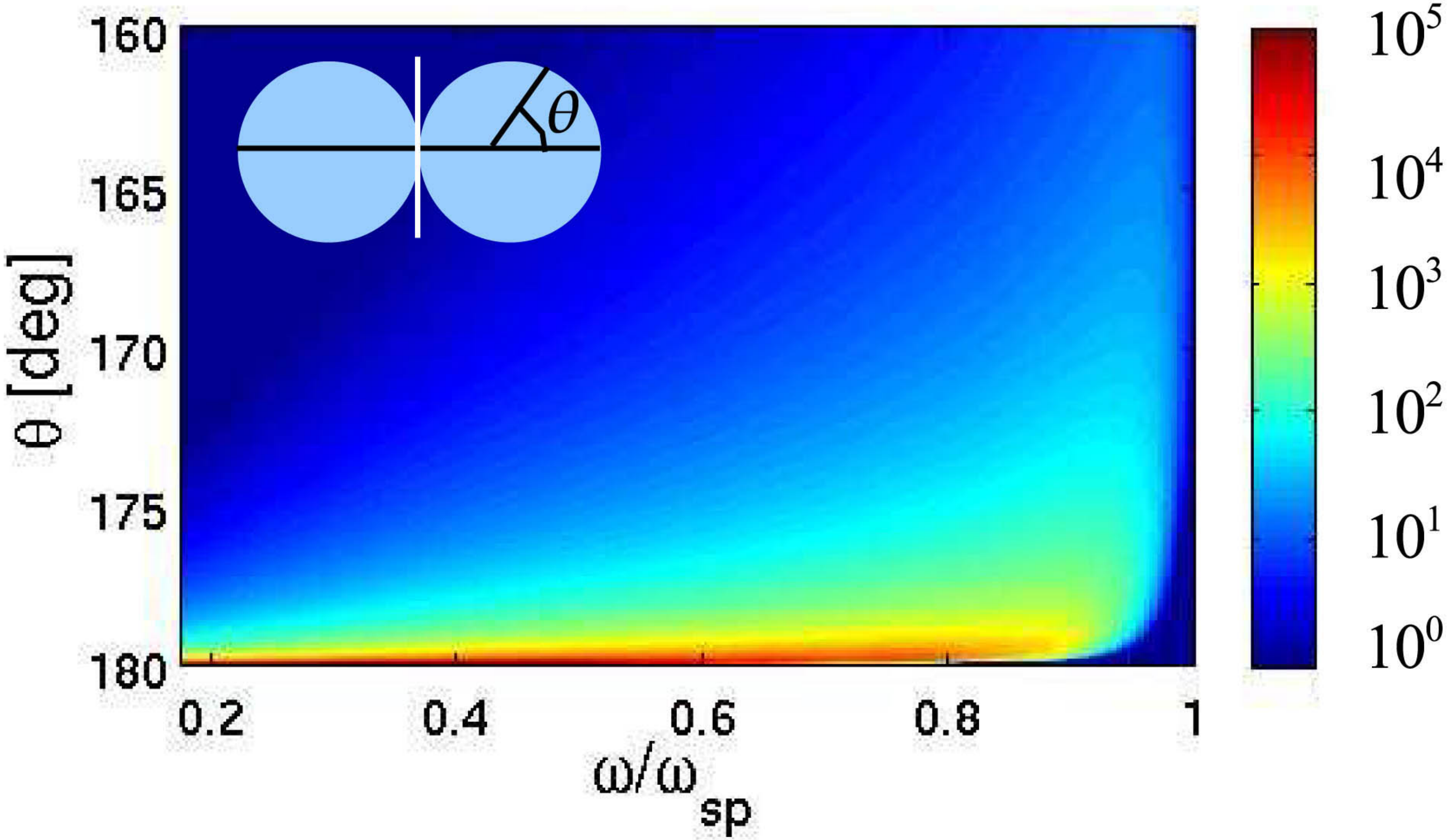
Calculated E_x normalized to the incoming field (E-field along x). The left and right panels display the field in the crescent and in the two kissing cylinders respectively. The metal is silver and $\omega = 0.9\omega_{sp}$. The scale is restricted to -10^{+5} to $+10^{+5}$ but note that the field magnitude is far larger around the structural singularities.

Field enhancement versus angle – crescent



Blue curve: E_x at the surface of the crescent, plotted as a function of θ, for $\omega = 0.75\omega_{sp}$ and $\varepsilon = -7.058 + 0.213i$ taken from Johnson and Christy. **Red curve:** $\varepsilon = -7.058 + 2 \times 0.213i$ i.e. more loss. Both curves are normalised to the incoming field amplitude $E0$. The crescent is defined by the ratio of diameters $r = 0.5$

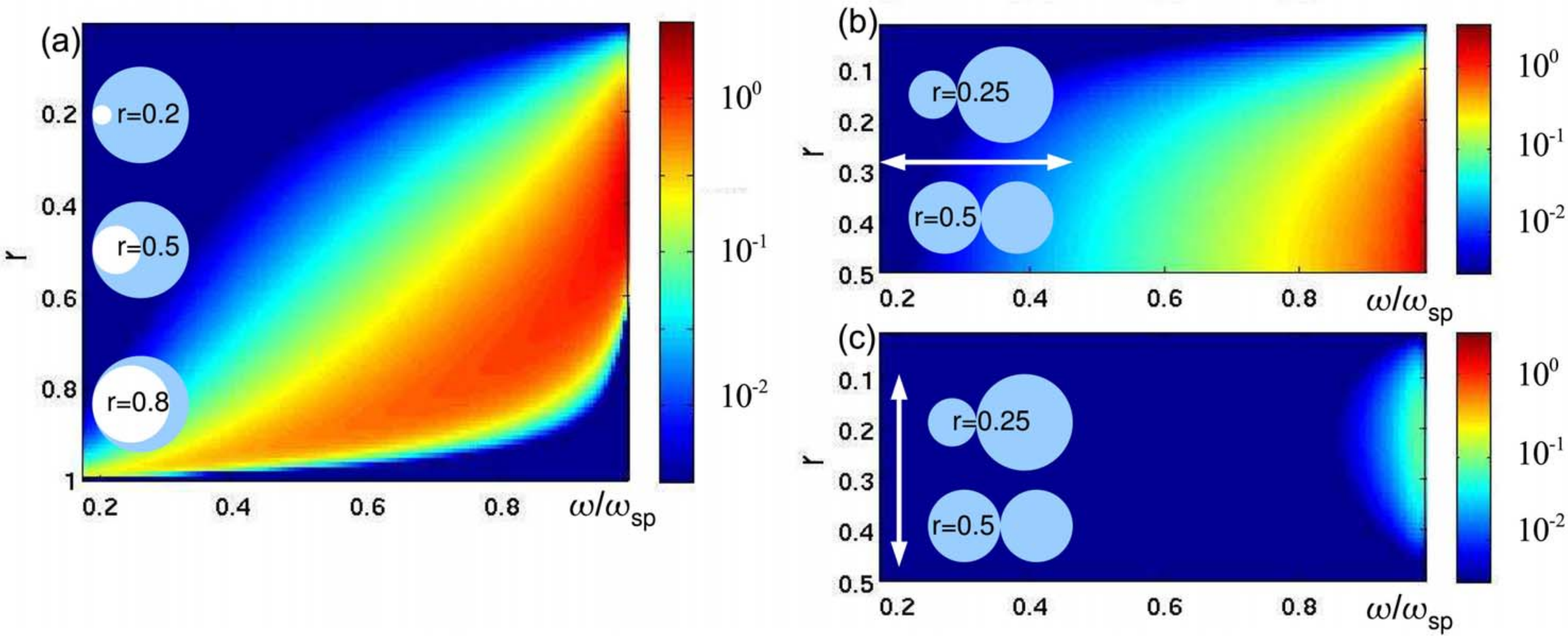
Field enhancement versus angle – kissing cylinders



Field enhancement, $|E|/E_0$, along the cylinder surface as a function of θ and ω for a plane wave incident normal to the axis of the cylinders. The two kissing cylinders are of same size (r = 0.5).

Absorption cross-section

as a fraction of the physical cross-section as a function of r and ω



Left for the crescent and **right** for the kissing cylinders with an incident electric field polarized along x (b) and along y (c).

The overall size of each device is 20 nm.

Benefits of Broadband Harvesting

- Multi - frequency systems benefit from enhancement of all frequencies.
- e.g. one system can enhance detection of a wide range of molecules
- e.g. if we are amplifying a weak signal at one frequency using a pump of another frequency, both are enhanced.

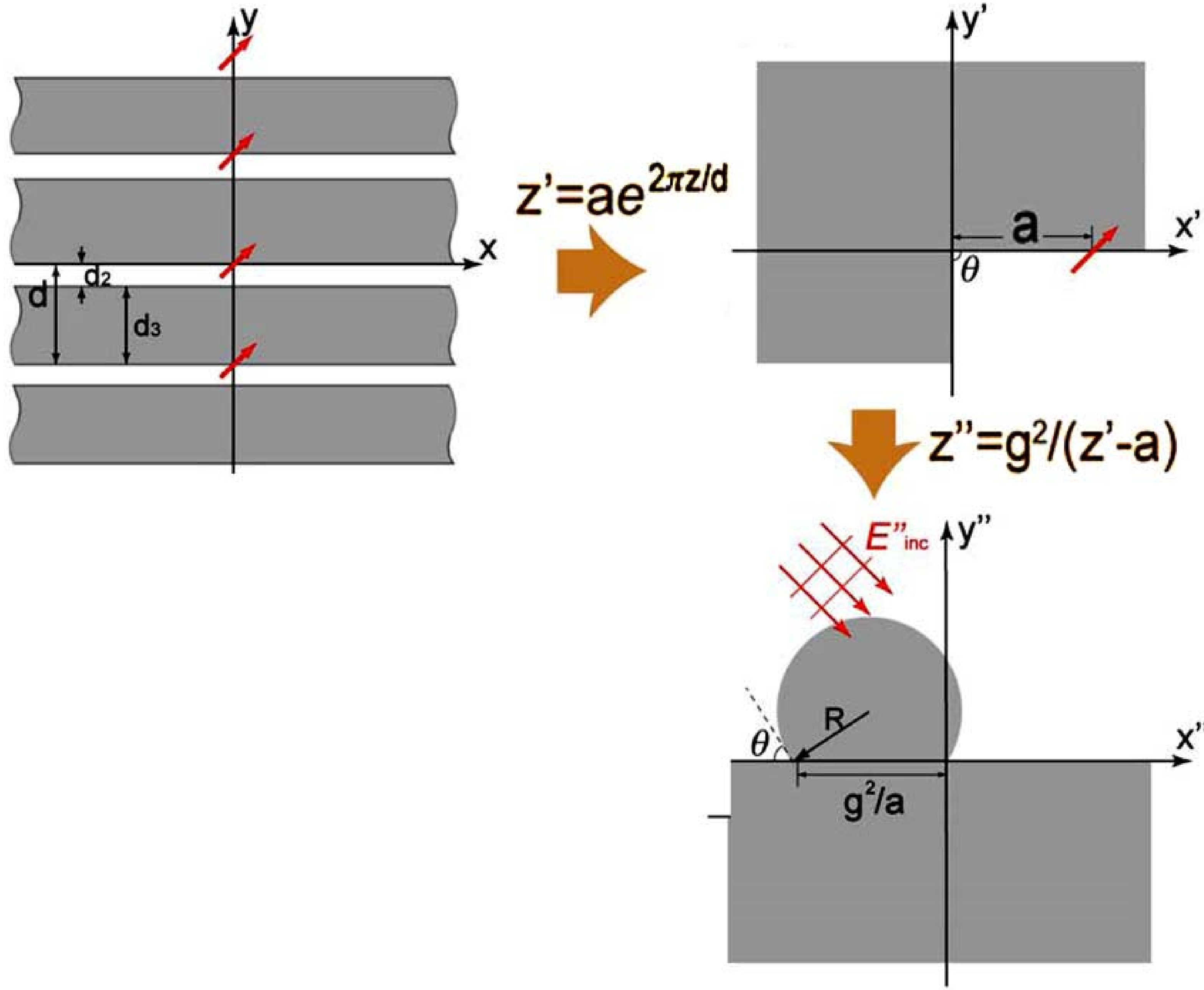
What can go wrong?

The theory predicts spectacular enhancements in the harvested fields, even when realistic values of the silver permittivity are included. Enhancements in field strength of 10^4 are predicted, implying an enhancement of the SERS signal of 10^{16}. Several factors will prevent this ideal from being attained:

- radiative losses
- non locality of ε
- problems in nm scale precision manufacture

Nevertheless substantial effects can be expected

Other Related Systems *(work by Yu Luo)*



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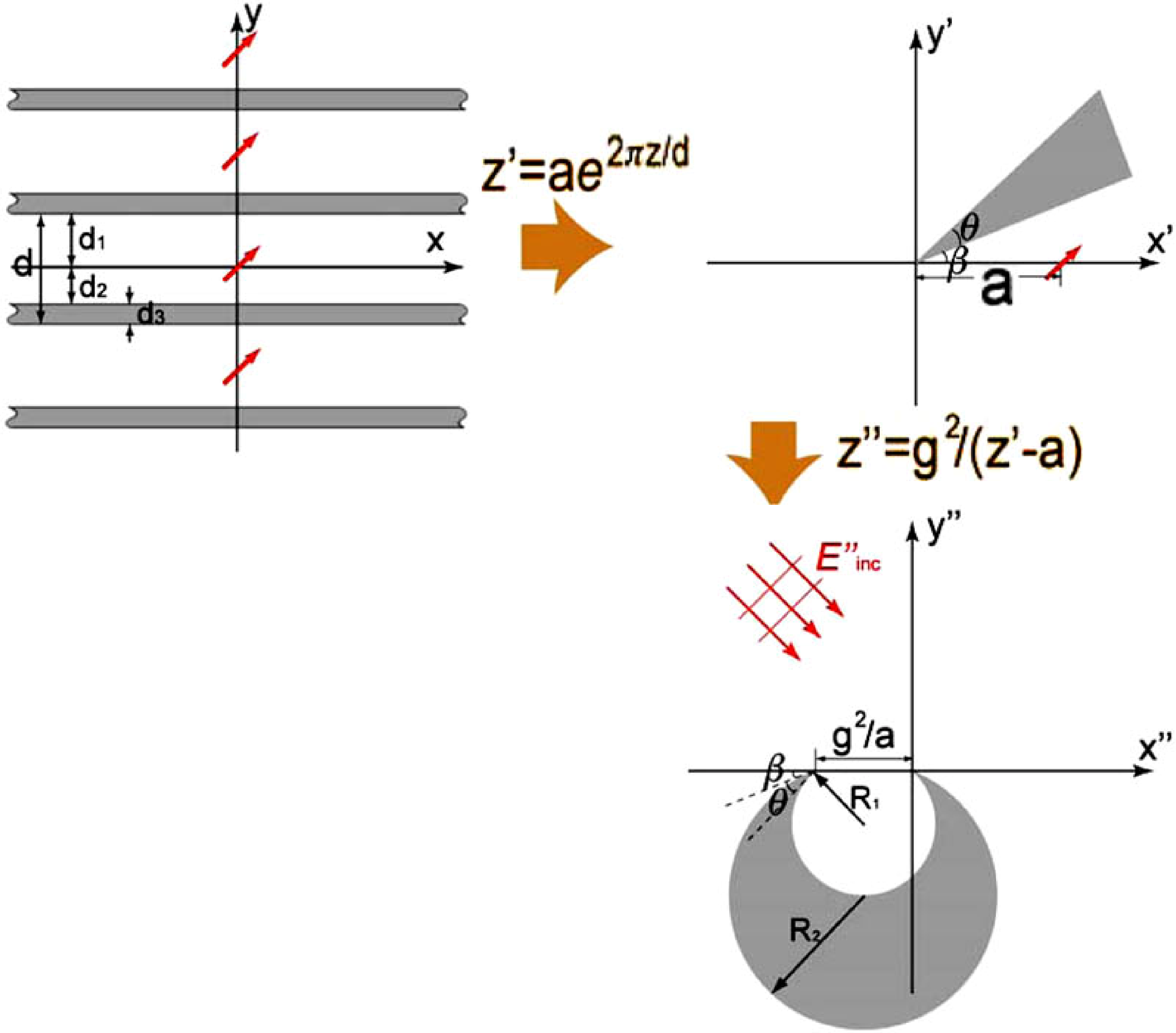

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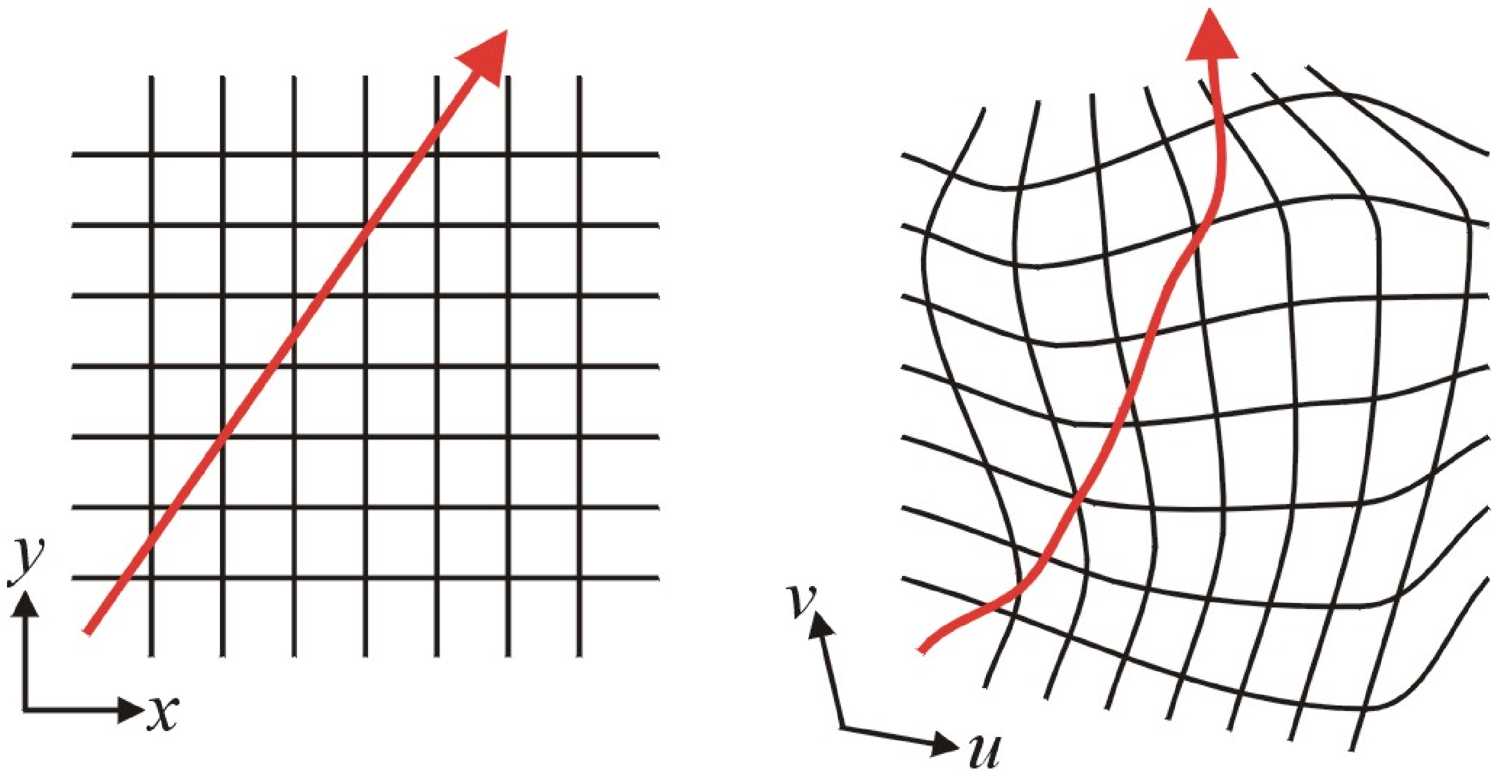
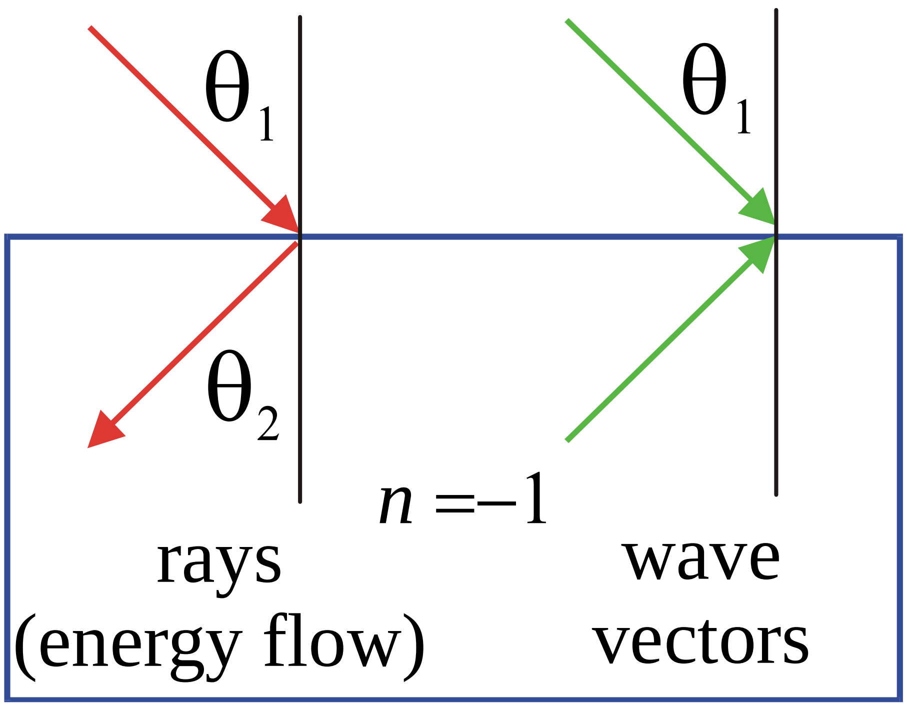


Left: a field line in free space with the background Cartesian coordinate grid shown. Right: the distorted field line with the background coordinates distorted in the same fashion.

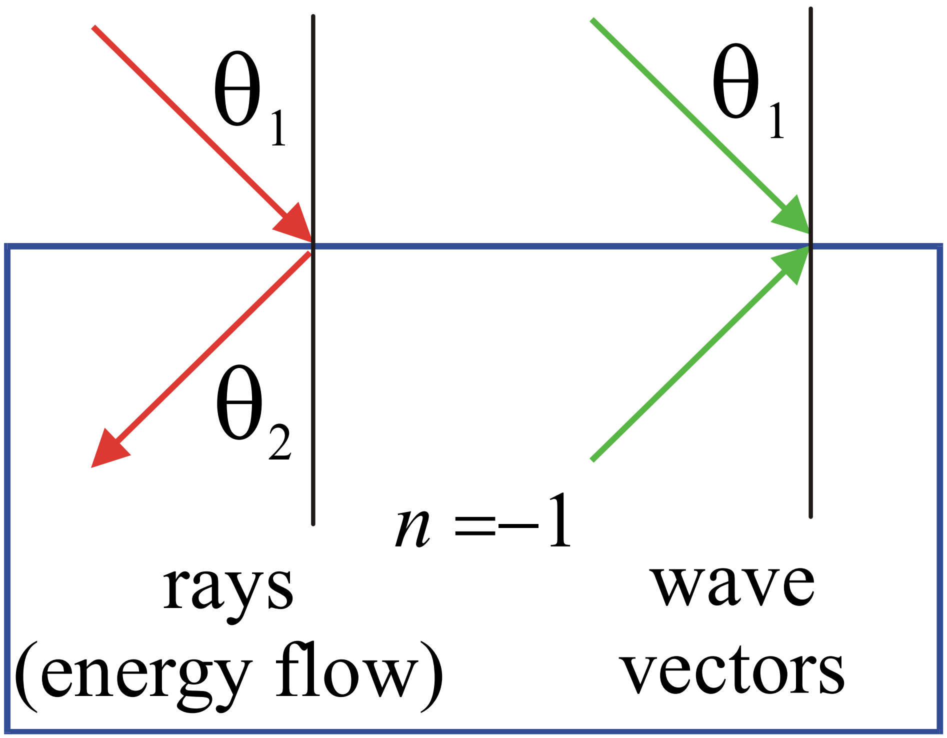
Negative Refractive Index and Snell's Law

$$n = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

Hence in a negative refractive index material, *light makes a negative angle with the normal*. Note that the parallel component of wave vector is always preserved in transmission, but that energy flow is opposite to the wave vector.



The consequences of negative refraction
1. negative group velocity



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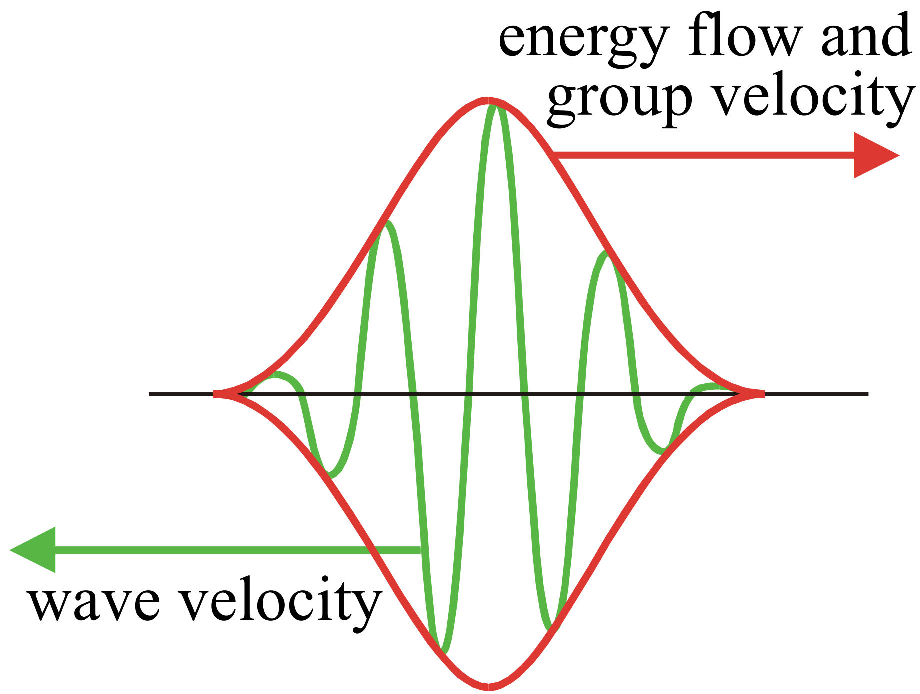
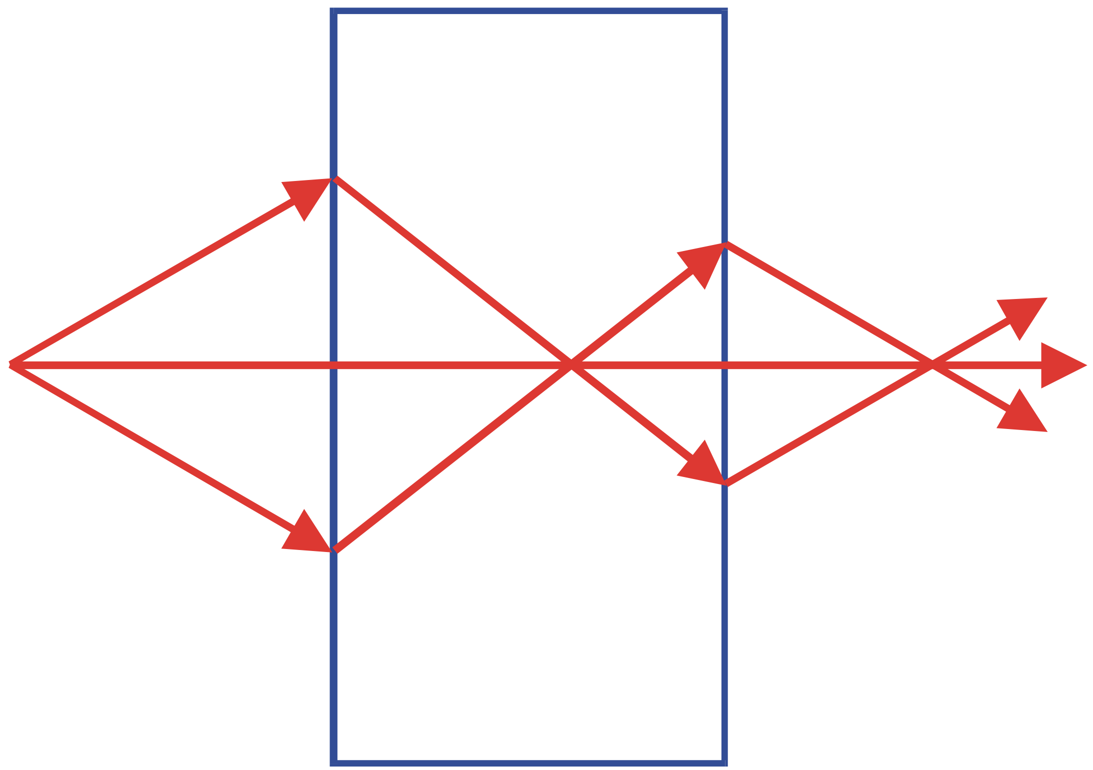


Materials with negative refraction are sometimes called ***left handed materials*** because the Poynting vector has the opposite sign to the wave vector.

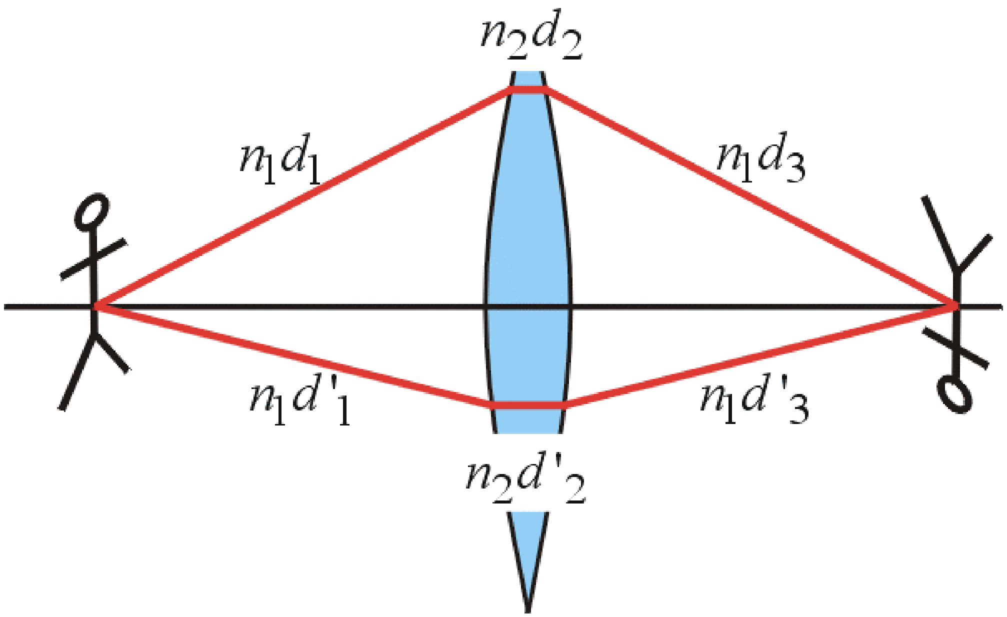
Negative Refractive Index and Focussing



A negative refractive index medium bends light to a negative angle relative to the surface normal. Light formerly diverging from a point source is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time.

Fermat's Principle:

"Light takes the shortest optical path between two points"

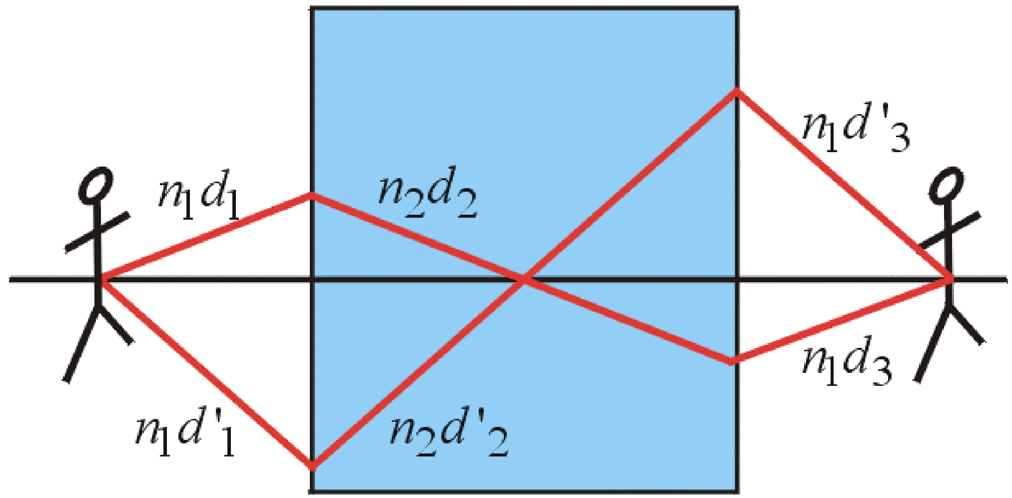


e.g. for a lens the shortest optical distance between object and image is:

$$n_1d_1 + n_2d_2 + n_1d_3 = n_1d'_1 + n_2d'_2 + n_1d'_3$$

both paths converge at the same point because both correspond to a minimum.

If n_2 is negative the ray traverses **negative optical space**.



for a *perfect* lens the shortest optical distance between object and image is **zero**:

$$0 = n_1d_1 + n_2d_2 + n_1d_3$$
$$= n_1d'_1 + n_2d'_2 + n_1d'_3$$

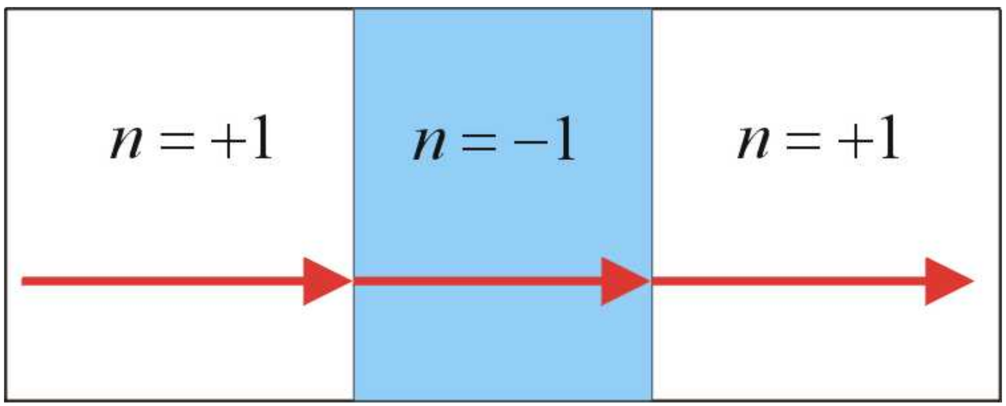
For a perfect lens the image *is* the object

Can we make negative space in the laboratory?

According to Einstein, negative space can be modelled by a negative refractive index.

What does light do in negative space?

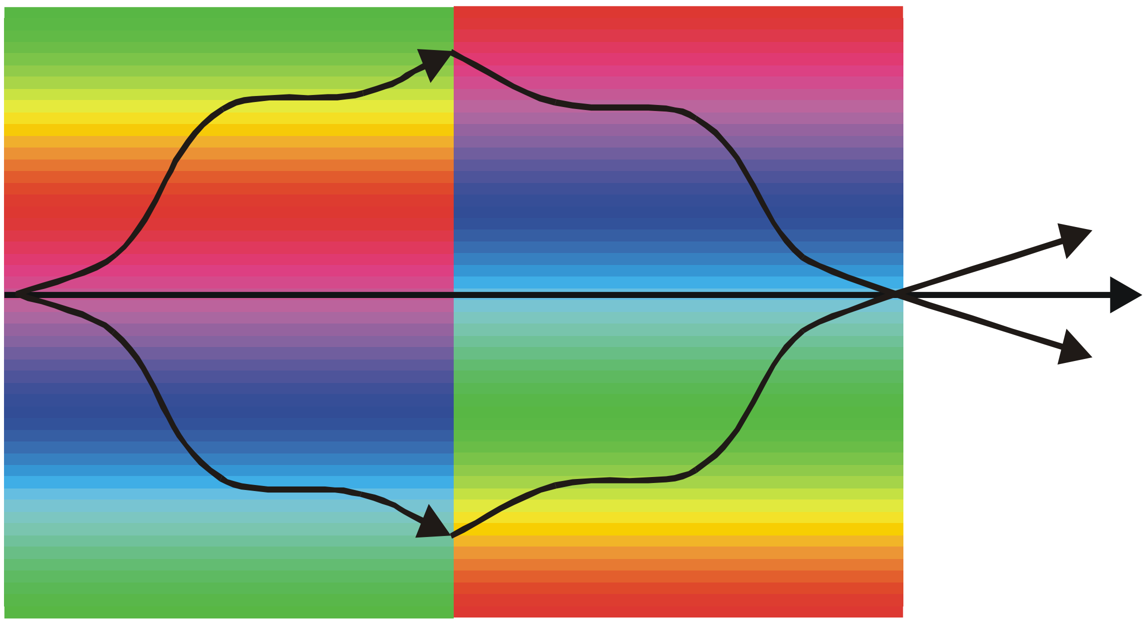
This is what happens in real space:



This is how light sees the world:

Negative Space

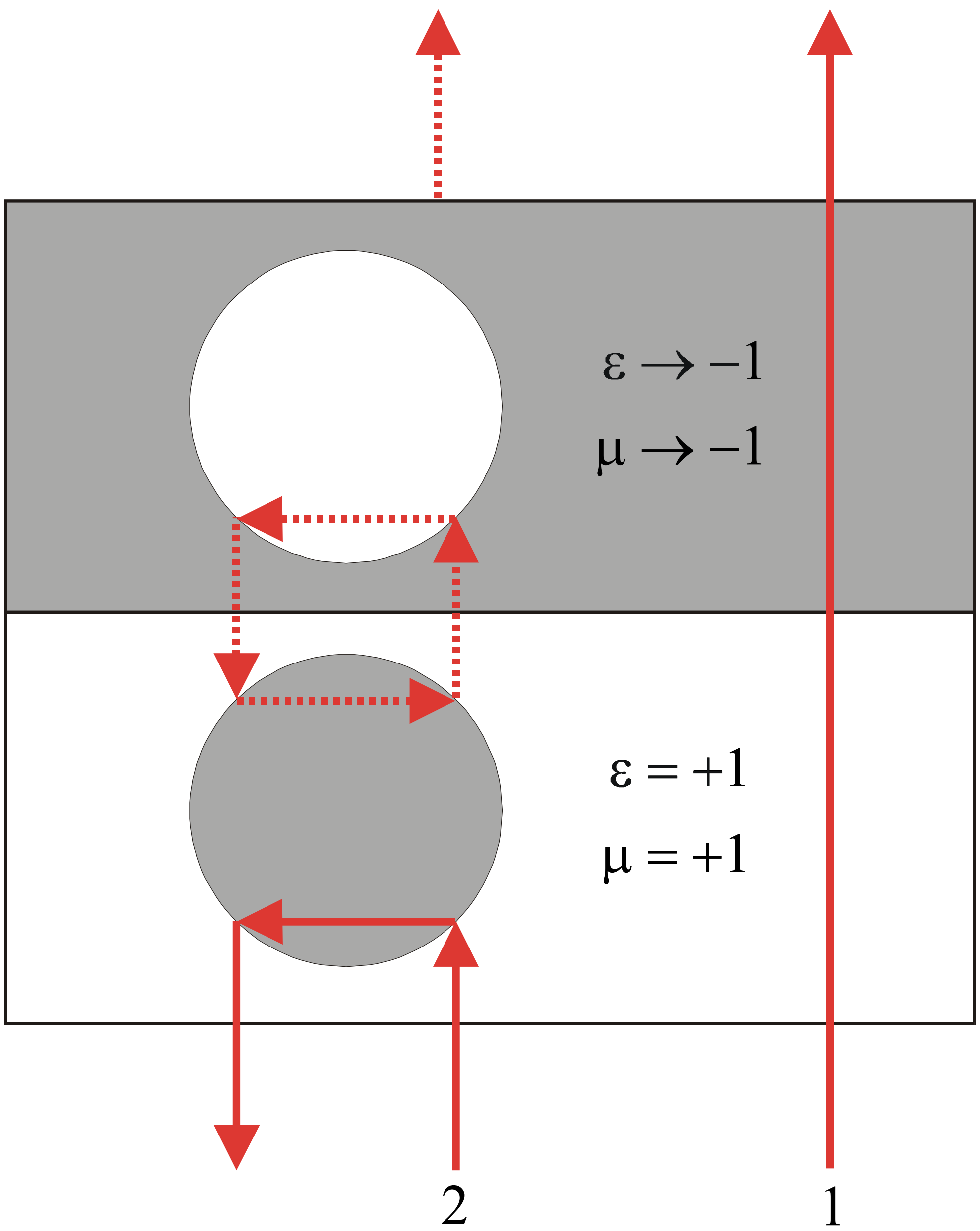
A slab of $n = -1$ material thickness d, cancels the effect of an equivalent thickness of free space. i.e. objects are focussed a distance $2d$ away. An alternative pair of complementary media, each cancelling the effect of the other. The light does not necessarily follow a straight line path in each medium:



General rule:
two regions of space optically cancel if in each region ε,μ are reversed mirror images.

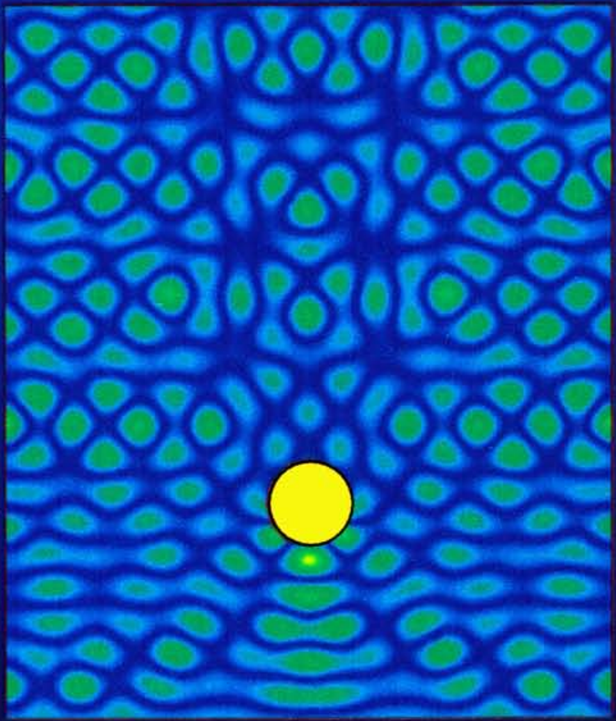
The overall effect is as if a section of space thickness $2d$ were removed from the experiment.

A Negative Paradox

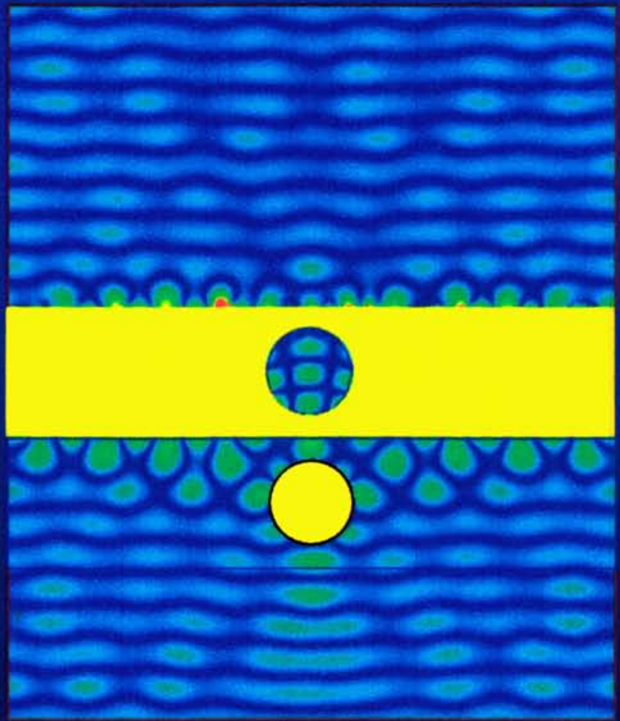


The left and right media in this 2D system are negative mirror images and therefore optically annihilate one another. However a ray construction appears to contradict this result. Nevertheless the theorem is correct and the ray construction erroneous. Note the closed loop of rays indicating the presence of resonances.

Compensation of inhomogeneous media



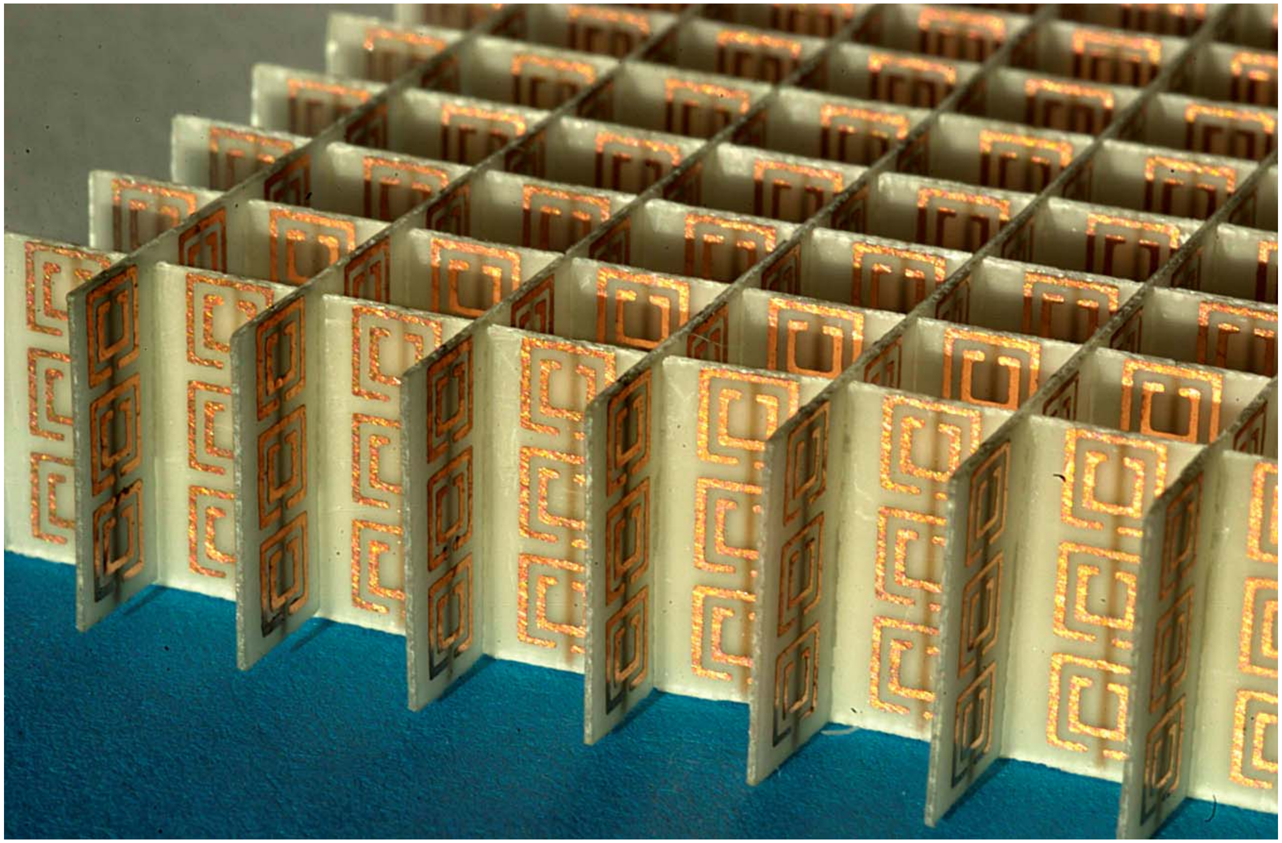
Scattering from a cylinder with n=-1.4



Compensation of the n=-1.4 cylinder

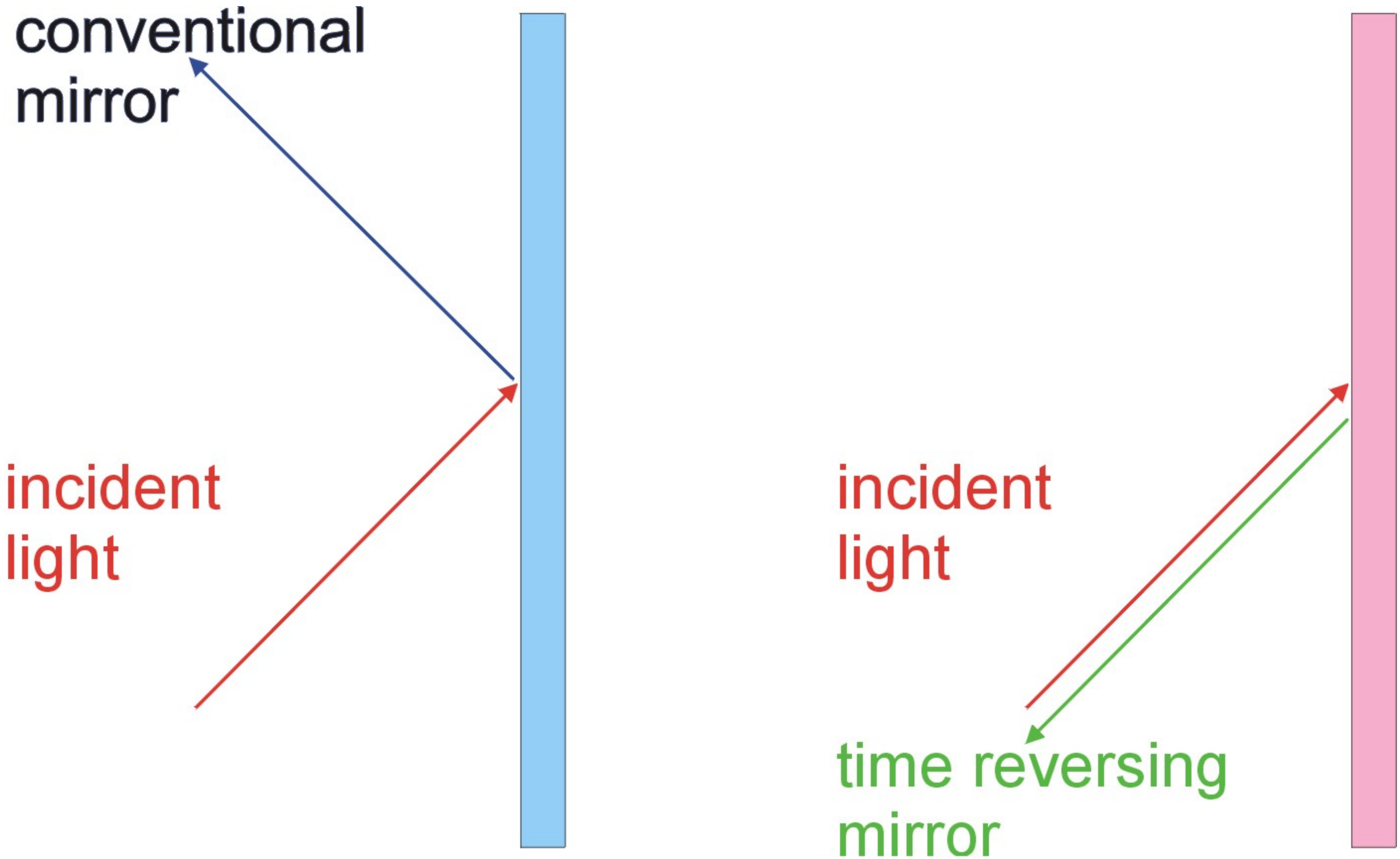
Negative Refraction at Optical Frequencies

At microwave frequencies *metamaterials* can be used to create negative refraction in the laboratory.

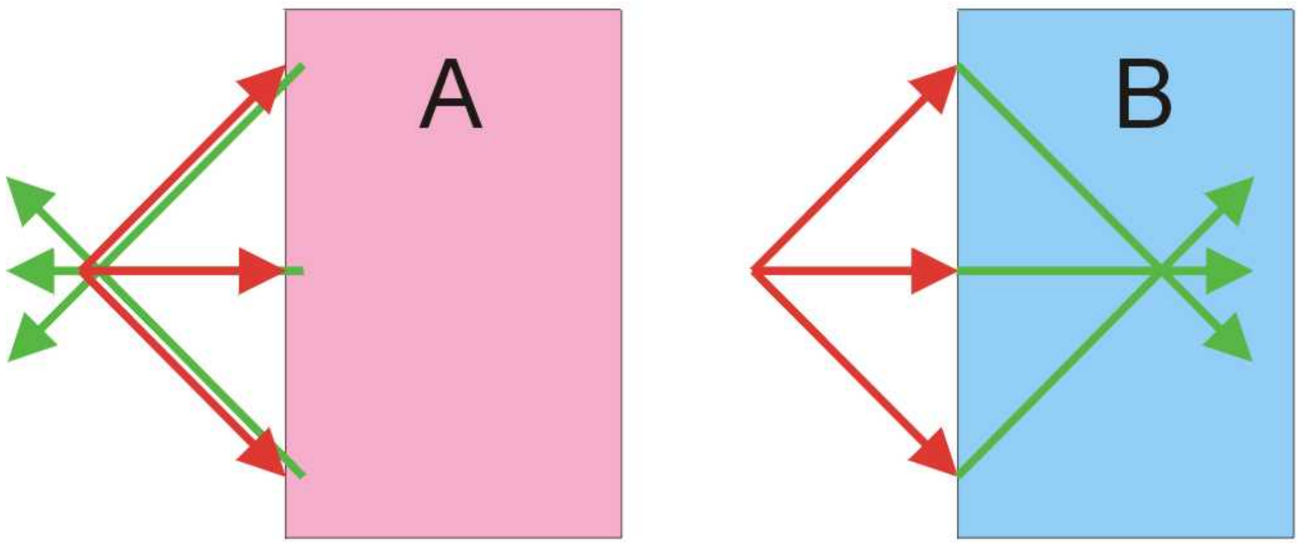


However this approach of creating function through structure is a challenge at higher frequencies. Creating negative refraction is particularly difficult.

Time reversal – what is it?



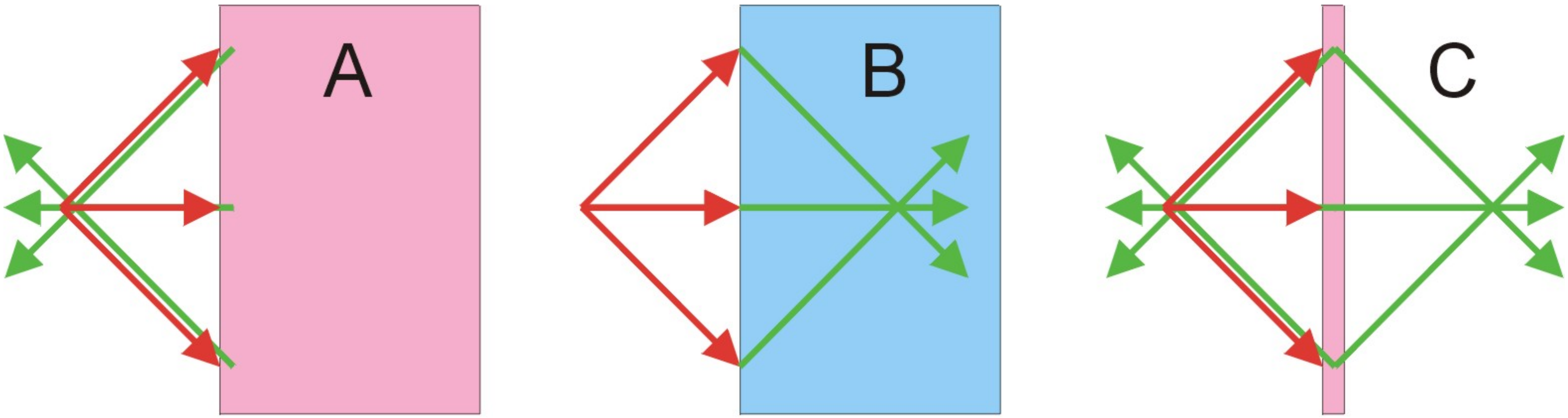
Similarities between time reversal and negative refraction



time reversal

negative refraction

Similarities between time reversal and negative refraction



time reversal | negative refraction | 2D time reversal

S. Maslovski and S. Tretyakov, "Phase conjugation and perfect lensing"
J. Appl. Phys., **94**, 4241 (2003)

K. Kobayashi, *J. Phys. Condens. Matter* **18**, 3703 (2006)
"Complementary media of electrons [in graphene]"

J.B. Pendry, "Time Reversal & Negative Refraction",
Science **322** 71 (2008)

Time reversal is really *frequency* reversal

forward travelling wave:

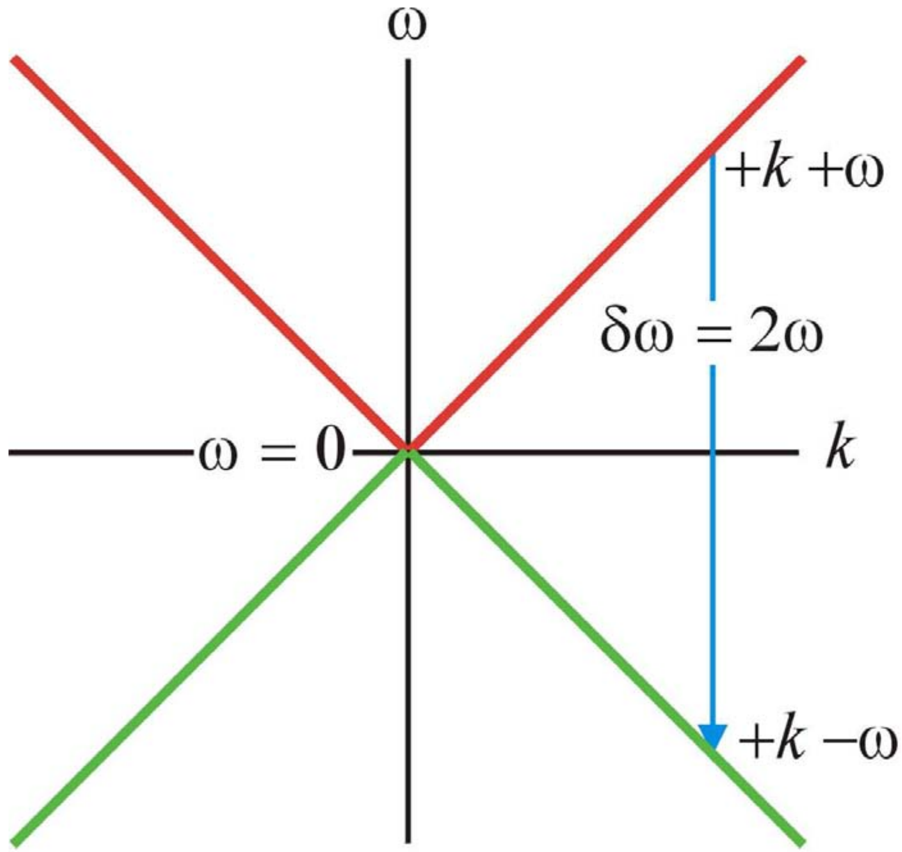
$$\exp i\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)$$

reverse the frequency, $+\omega \rightarrow -\omega$

$$\exp i\left(\mathbf{k}\cdot\mathbf{r}+\omega t\right)$$

and the wave reverses its direction.

Time reversal is really *frequency* reversal



Maxwell's equation support both positive and negative frequency solutions.

Time reversal can be viewed as a vertical transition from +ω to –ω.

Note that for negative frequencies $\boxed{v_g \times v_p < 0}$ - the signature of negative refraction!

Self-conjugate media

are defined as containing both positive and negative refracting states but at difference frequencies.

All dielectric media are self conjugate between $\pm\,\omega$.

On the right are four instances of refraction: only when the frequency changes at the interface do we observe a negative angle of refraction.

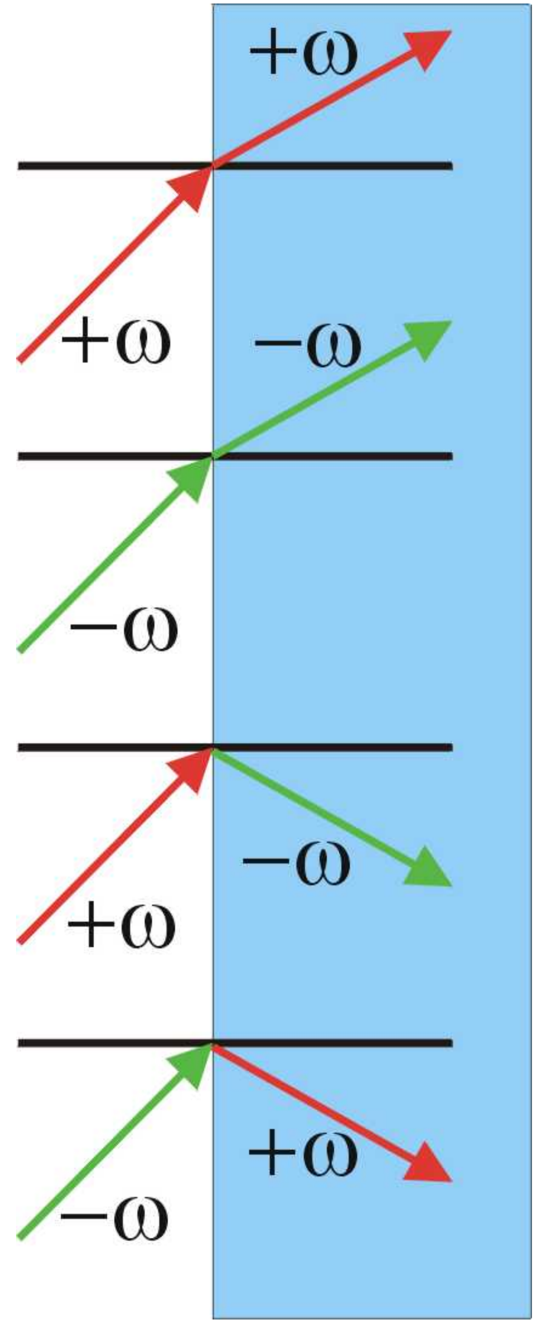
reminder: Snell's law contains the *ratio* of indices:

$$\frac{n_2}{n_1} = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

Hence by flipping the frequency at an interface we can simulate the effect of negative refraction.

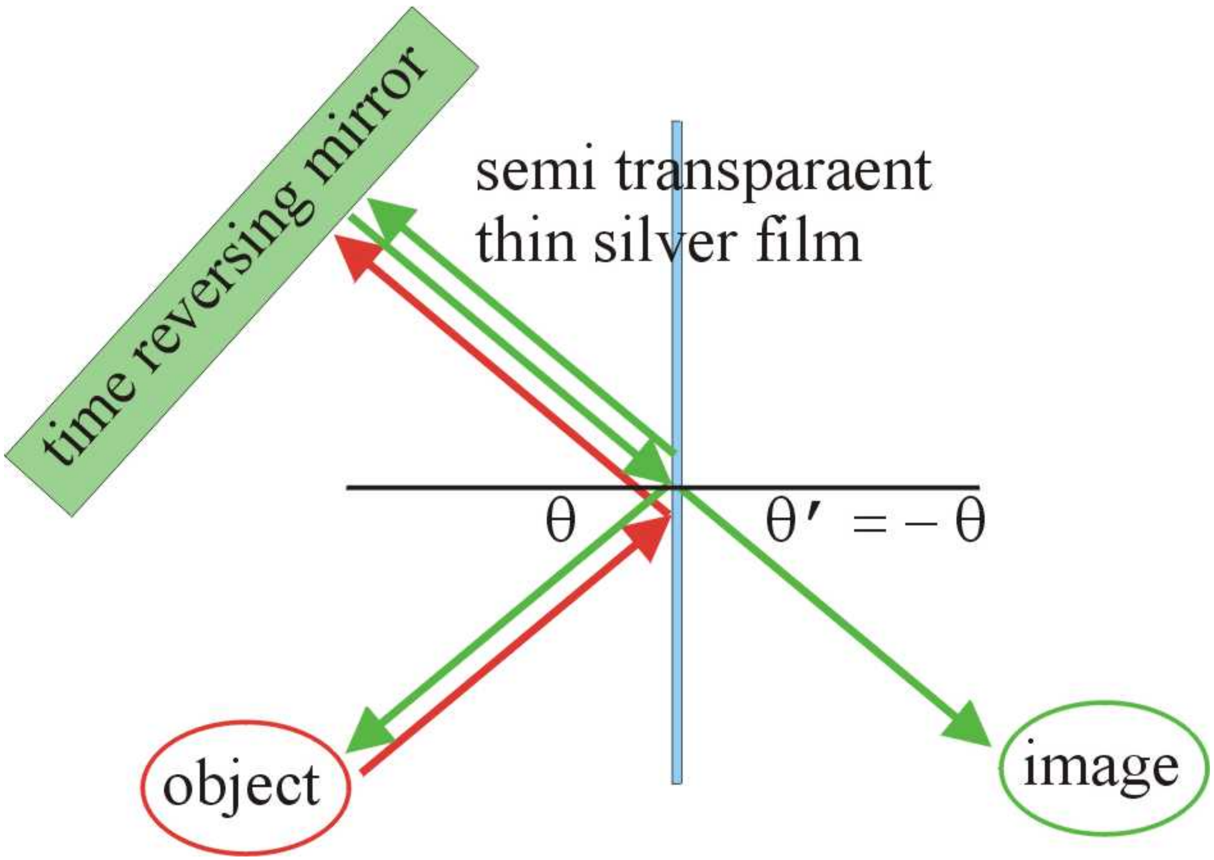
The most perfect negative refraction medium is the vacuum

see: K. Kobayashi, *J. Phys. Condens. Matter* **18**, 3703 (2006) "Complementary media of electrons [in graphene]"



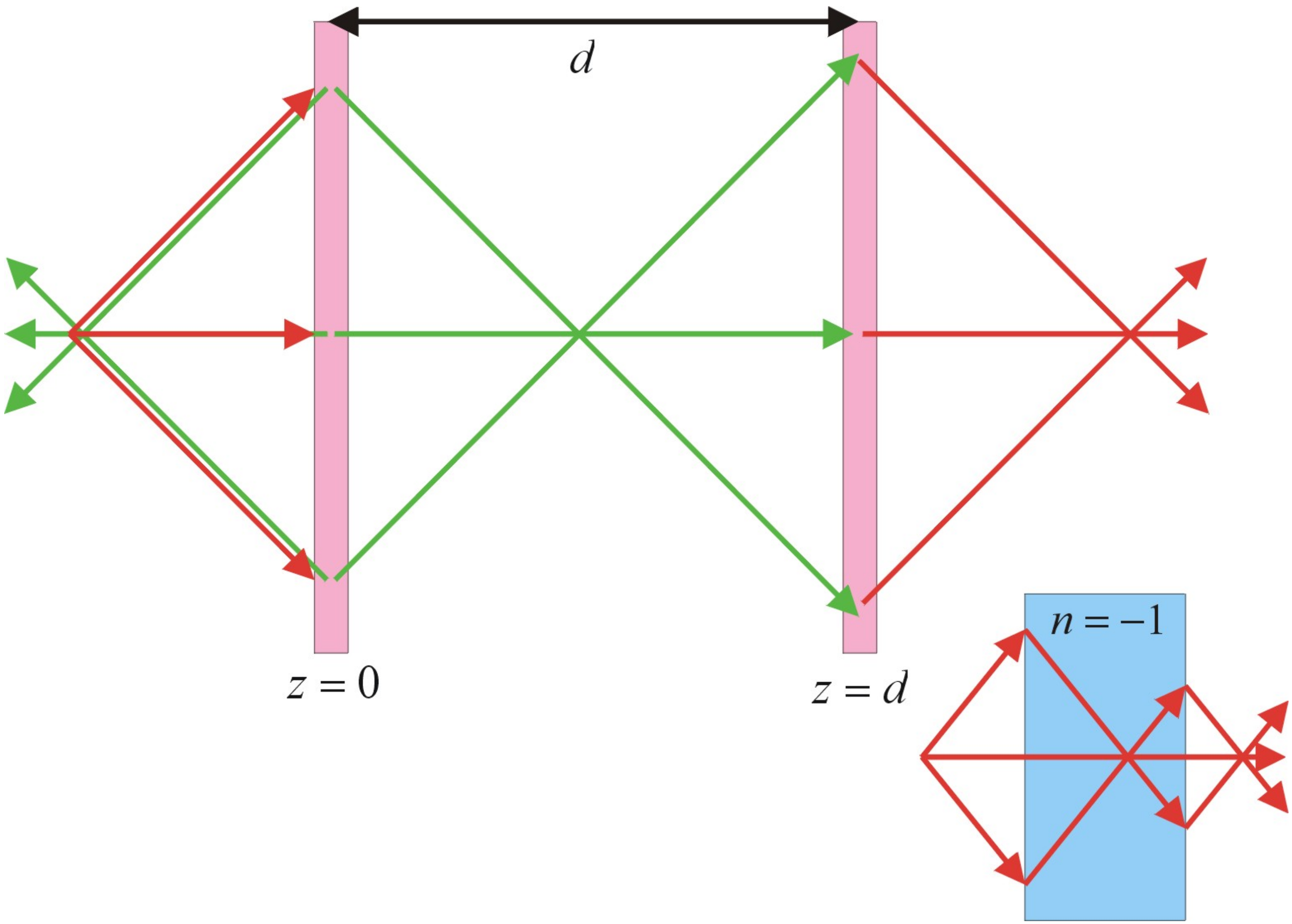
Time Reversal & Negative Refraction

Time reversal combined with a single reflection (and one transmission) at a half silvered mirror, reproduces the effect of negative refraction with an index of $n = -1$.



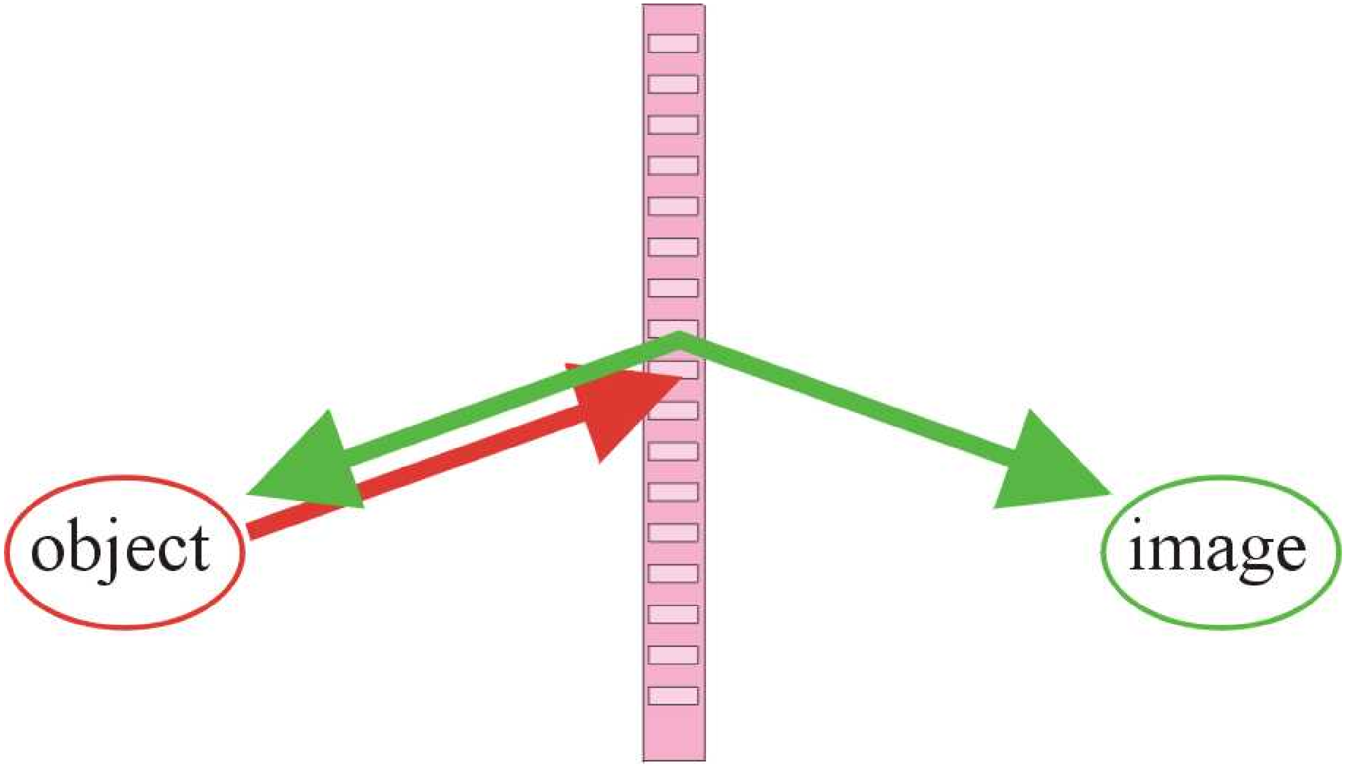
We can use this trick to create the effect of an interface of any shape between positively and negatively refracting media, whilst in reality using only positively refracting materials.

Blueprint for a low loss lens at optical frequencies



Two frequency reversal sheets flip the rays into negatively refracting states simulating the surfaces of a negatively refracting material and focussing light.

The missing ingredient



2D hologram

To generate the negative refracting beams time reversal must take place in a thin sheet of material, thickness << wavelength.

To succeed we need a strongly non linear metamaterial.

Transformation Optics
& the Control of Electromagnetic Radiation

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Abstract: Ray optics gives us some control over propagation of light, but fails to account for the wave nature of light. Even more spectacularly it has nothing at all to say about controlling the so called ‘near field’. In contrast the new technique of transformation optics offers the possibility of complete control of radiation, correct to the level of Maxwell’s equations. Using this technology we can specify the material parameters needed to arrange the electric fields, magnetic fields and the Poynting vector almost as we choose.

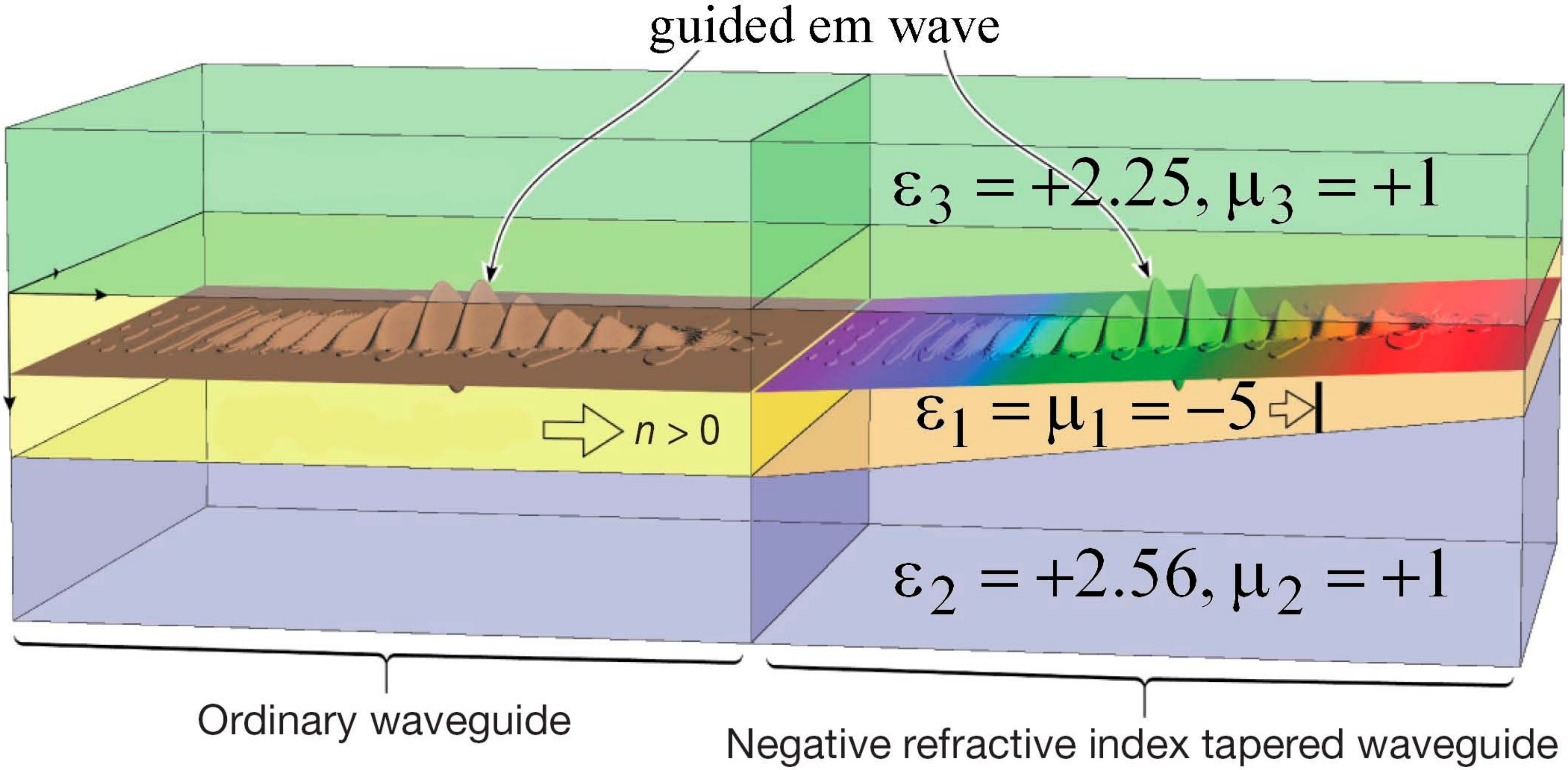
D. Schurig, J.B. Pendry, D.R. Smith, *Optics Express* **14**, 9794 (2006).

J.B. Pendry, *Contemporary Physics* **45**, 191 (2004).

J.B. Pendry in *“Coherence and Quantum Optics IX”*, ed. P. Bigelow, J.H. Eberly and C.R. Stroud, Jr. (OSA Publications), pp. 42-52. (2009).

The ‘trapped rainbow’

KL Tsakmakidis, AD. Boardman & O Hess, Nature **450** 397 (2007)



Different frequency components of a guided wave packet *stop without being reflected* at different thicknesses inside a tapered left-handed waveguide.